

Systems of Three Equations

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

1. Solve by elimination.

$x + y + z = 2$
 $x + 2y + 2z = 5$
 $x + 3y = 2$

2. Solve by elimination.

$x + 2y + 2z = -2$
 $x + 4y + z = -1$
 $x + 6y = 1$

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Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

3. Solve by elimination.

$x + y + z = 2$
 $2x + 2y + 2z = 4$
 $-x - y - z = -2$

4. Solve by elimination.

$x + z = 3$
 $x + 2y = 4$
 $x + y + 3z = 7$

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Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

5. Solve by elimination.

$x - 2y + z = 2$
 $x - y + 3z = 5$
 $x - 5y - 5z = -6$

6. Solve by elimination.

$x + 2y + 2z = -2$
 $x + 4y + z = -1$
 $x + 6y = 0$

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

1. One solution

Method: Elimination

$$R2 \leftarrow R2 + -1 R1: x + y + z = 2; y + z = 3; x + 3y = 2$$

$$R3 \leftarrow R3 + -1 R1: x + y + z = 2; y + z = 3; 2y - z = 0$$

$$R3 \leftarrow R3 + -2 R2: x + y + z = 2; y + z = 3; -3z = -6$$

$$R3 \leftarrow (-1/3) R3: x + y + z = 2; y + z = 3; z = 2$$

Back-substitute in the last triangular equation: $z = 2$

Back-substitute in the middle triangular equation: $y = 1$

Back-substitute in the first triangular equation: $x = -1$

Back-substitute: $(x, y, z) = (-1, 1, 2)$

$$\text{Check E1: } 2 = 2$$

$$\text{Check E2: } 5 = 5$$

$$\text{Check E3: } 2 = 2$$

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

2. No solution

Method: Elimination

$$R2 \leftarrow R2 + -1 R1: x + 2y + 2z = -2; 2y - z = 1; x + 6y = 1$$

$$R3 \leftarrow R3 + -1 R1: x + 2y + 2z = -2; 2y - z = 1; 4y - 2z = 3$$

$$R2 \leftarrow (1/2) R2: x + 2y + 2z = -2; y - (1/2)z = (1/2); 4y - 2z = 3$$

$$R3 \leftarrow R3 + -4 R2: x + 2y + 2z = -2; y - (1/2)z = (1/2); 0 = 1$$

Contradiction: $0 = 1$ (nonzero), so there is no solution.

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

3. Infinitely many solutions

Method: Elimination

$R2 \leftarrow R2 + -2 R1$: $x + y + z = 2$; $0 = 0$; $-x - y - z = -2$

$R3 \leftarrow R3 + 1 R1$: $x + y + z = 2$; $0 = 0$; $0 = 0$

Dependent equation: $0 = 0$. Free variables: s , t .

Let $s = y$ and $t = z$; then $x = 2 + -1s + -1t$.

$(x, y, z) = (2, 0, 0) + s(-1, 1, 0) + t(-1, 0, 1)$

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

4. One solution

Method: Elimination

$$R2 \leftarrow R2 + -1 R1: x + z = 3; 2y - z = 1; x + y + 3z = 7$$

$$R3 \leftarrow R3 + -1 R1: x + z = 3; 2y - z = 1; y + 2z = 4$$

$$R2 \leftarrow (1/2) R2: x + z = 3; y - (1/2)z = (1/2); y + 2z = 4$$

$$R3 \leftarrow R3 + -1 R2: x + z = 3; y - (1/2)z = (1/2); (5/2)z = (7/2)$$

$$R3 \leftarrow (2/5) R3: x + z = 3; y - (1/2)z = (1/2); z = (7/5)$$

$$\text{Back-substitute in the last triangular equation: } z = (7/5)$$

$$\text{Back-substitute in the middle triangular equation: } y = (6/5)$$

$$\text{Back-substitute in the first triangular equation: } x = (8/5)$$

$$\text{Back-substitute: } (x, y, z) = ((8/5), (6/5), (7/5))$$

$$\text{Check E1: } 3 = 3$$

$$\text{Check E2: } 4 = 4$$

$$\text{Check E3: } 7 = 7$$

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

5. No solution

Method: Elimination
R2 <- R2 + -1 R1: $x - 2y + z = 2$; $y + 2z = 3$; $x - 5y - 5z = -6$
R3 <- R3 + -1 R1: $x - 2y + z = 2$; $y + 2z = 3$; $-3y - 6z = -8$
R3 <- R3 + 3 R2: $x - 2y + z = 2$; $y + 2z = 3$; $0 = 1$
Contradiction: $0 = 1$ (nonzero), so there is no solution.

Systems of Three Equations

Worked Answer Key

Solve each three-equation system with the selected method. Give an exact ordered triple, a complete parameterization, or a clear contradiction.

6. Infinitely many solutions

Method: Elimination

$$R2 \leftarrow R2 + -1 R1: x + 2y + 2z = -2; 2y - z = 1; x + 6y = 0$$

$$R3 \leftarrow R3 + -1 R1: x + 2y + 2z = -2; 2y - z = 1; 4y - 2z = 2$$

$$R2 \leftarrow (1/2) R2: x + 2y + 2z = -2; y - (1/2)z = (1/2); 4y - 2z = 2$$

$$R3 \leftarrow R3 + -4 R2: x + 2y + 2z = -2; y - (1/2)z = (1/2); 0 = 0$$

Dependent equation: $0 = 0$. Free variable: t .

Let $t = z$; then $y = (1/2) + (1/2)t$ and $x = -3 + -3t$.

$$(x, y, z) = (-3, (1/2), 0) + t(-3, (1/2), 1)$$