

Z-Scores and Standardized Values

Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

1. Data set A: $x = 52.75$, mean = 48, SD = 9.5 points. Data set B: $x = 23$, mean = 36, SD = 6.5 centimeters. Which value has the greater standardized position?

Set	x	mean	SD	unit
A	52.75	48	9.5	points
B	23	36	6.5	centimeters

2. Data set A measurement has mean 89 points and standard deviation 11 points. Find x when $z = 0.5$.

Set	x	mean	SD	unit
A	?	89	11	points

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Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

3. Data set A measurement has mean 86.5 minutes and standard deviation 9.5 minutes. Find the z-score for 81.75 minutes.

Set	x	mean	SD	unit
A	81.75	86.5	9.5	minutes

4. Data set A: $x = 71$, mean = 57.5, SD = 13.5 degrees. Data set B: $x = 29.75$, mean = 45.5, SD = 10.5 minutes. Which value has the greater standardized position?

Set	x	mean	SD	unit
A	71	57.5	13.5	degrees
B	29.75	45.5	10.5	minutes

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Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

5. Data set A measurement has mean 51 points and standard deviation 10 points. Find x when z = 0.5.

Set	x	mean	SD	unit
A	?	51	10	points

6. Data set A measurement has mean 29 points and standard deviation 2.5 points. Find the z-score for 30.25 points.

Set	x	mean	SD	unit
A	30.25	29	2.5	points

Z-Scores and Standardized Values

Worked Answer Key

Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

1. Data set A: $x = 52.75$, mean = 48, SD = 9.5 points. Data set B: $x = 23$, mean = 36, SD = 6.5 centimeters. Which value has the greater standardized position?

Set	x	mean	SD	unit
A	52.75	48	9.5	points
B	23	36	6.5	centimeters

Answer: Data set A has the greater standardized position ($z = 0.5$)

1. Data set A: $z = (52.75 - 48) / 9.5 = 0.5$.

2. Data set B: $z = (23 - 36) / 6.5 = -2$.

3. Data set A is farther above, or less far below, its own mean. A greater standardized position is not automatically a better outcome; context would determine that.

2. Data set A measurement has mean 89 points and standard deviation 11 points. Find x when $z = 0.5$.

Set	x	mean	SD	unit
A	94.5	89	11	points

Answer: $x = 94.5$ points; it is 0.5 standard deviations above the mean

1. $x = \text{mean} + z(\text{standard deviation})$

2. $x = 89 + 0.5(11) = 94.5$ points.

3. The sign of z shows whether x is above or below its own mean.

Z-Scores and Standardized Values

Worked Answer Key

Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

3. Data set A measurement has mean 86.5 minutes and standard deviation 9.5 minutes. Find the z-score for 81.75 minutes.

Set	x	mean	SD	unit
A	81.75	86.5	9.5	minutes

Answer: $z = -0.5$; the value is 0.5 standard deviations below the mean

1. $z = (x - \text{mean}) / \text{standard deviation}$

2. $z = (81.75 - 86.5) / 9.5 = -0.5$.

3. A negative z-score places the value below its mean.

4. Data set A: $x = 71$, mean = 57.5, SD = 13.5 degrees. Data set B: $x = 29.75$, mean = 45.5, SD = 10.5 minutes. Which value has the greater standardized position?

Set	x	mean	SD	unit
A	71	57.5	13.5	degrees
B	29.75	45.5	10.5	minutes

Answer: Data set A has the greater standardized position ($z = 1$)

1. Data set A: $z = (71 - 57.5) / 13.5 = 1$.

2. Data set B: $z = (29.75 - 45.5) / 10.5 = -1.5$.

3. Data set A is farther above, or less far below, its own mean. A greater standardized position is not automatically a better outcome; context would determine that.

Z-Scores and Standardized Values

Worked Answer Key

Use $z = (x - \text{mean}) / \text{standard deviation}$. The given standard deviation is positive. Explain whether each value is above or below its mean; no normal-distribution assumption is needed.

5. Data set A measurement has mean 51 points and standard deviation 10 points. Find x when z = 0.5.

Set	x	mean	SD	unit
A	56	51	10	points

Answer: x = 56 points; it is 0.5 standard deviations above the mean

1. $x = \text{mean} + z(\text{standard deviation})$

2. $x = 51 + 0.5(10) = 56$ points.

3. The sign of z shows whether x is above or below its own mean.

6. Data set A measurement has mean 29 points and standard deviation 2.5 points. Find the z-score for 30.25 points.

Set	x	mean	SD	unit
A	30.25	29	2.5	points

Answer: z = 0.5; the value is 0.5 standard deviations above the mean

1. $z = (x - \text{mean}) / \text{standard deviation}$

2. $z = (30.25 - 29) / 2.5 = 0.5$.

3. A positive z-score places the value above its mean.