

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

1. Independent samples give group 1: $n=6$, $\text{mean}=14$, $s=2$; group 2: $n=9$, $\text{mean}=15$, $s=3$. Use the equal-variance pooled test of $H_0: \mu_1-\mu_2=0$ at $\alpha=0.1$.

Assumptions: Samples are independent and each population is approximately normal. Population variances are assumed equal.
Reference: Pool the variance and use $df=n_1+n_2-2$. t reference: $P(T_{13}>0.713229032)=0.2441552908$; two-sided $p=2(0.2441552908)=0.4883105816$. Critical $|t|=1.770933396$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

2. Matched before values are 13,16,15,19,17; after values are 14,18,14,22,20. Test whether the mean paired change differs from 0 at alpha=0.01.

Assumptions: Pairs are independent of other pairs; paired differences are approximately normal.

Reference: Use d=after-before, $t = \bar{d} / (s_d / \sqrt{n})$, $df = n - 1$. t reference: $P(T_4 \geq 2.138089935) = 0.0496503416$; two-sided $p = 2(0.0496503416) = 0.0993006832$. Critical $|t| = 4.604094871$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

3. A sample has $n=8$, $\bar{x}=16$, and $s=3$. Test $H_0: \mu=12$ against $H_A: \mu$ differs from 12 at $\alpha=0.05$.

Assumptions: The observations are independent and the population is approximately normal.
Reference: One-sample $t=(\bar{x}-\mu_0)/(s/\sqrt{n})$, $df=n-1$. t reference: $P(T_7 \geq 3.771236166)=0.0034858275$; two-sided $p=2(0.0034858275)=0.0069716550$. Critical $|t|=2.364624252$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

4. Independent samples give group 1: $n=9$, $\text{mean}=17$, $s=3.5$; group 2: $n=7$, $\text{mean}=15$, $s=7$. Use the unequal-variance Welch test of $H_0: \mu_1-\mu_2=0$ at $\alpha=0.1$.

Assumptions: Samples are independent and each population is approximately normal. Equal variances are not assumed.
Reference: Use $SE=\sqrt{s_1^2/n_1+s_2^2/n_2}$ and Welch-Satterthwaite df without rounding. t reference: $P(T_{8.324141812}\geq 0.691668501)=0.2539874797$; two-sided $p=2(0.2539874797)=0.5079749594$. Critical $|t|=1.850204075$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

5. Matched before values are 16,19,18,22,20; after values are 17,21,17,25,22. Test whether the mean paired change differs from 0 at alpha=0.01.

Assumptions: Pairs are independent of other pairs; paired differences are approximately normal.

Reference: Use d=after-before, $t = \frac{\bar{d} - \mu_0}{s_d / \sqrt{n}}$, df=n-1. t reference: $P(T_4 \geq 2.064187386) = 0.0539694112$; two-sided $p = 2(0.0539694112) = 0.1079388224$. Critical $|t| = 4.604094871$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

6. A sample has $n=6$, $\bar{x}=19$, and $s=2.5$. Test $H_0: \mu=10$ against $H_A: \mu$ differs from 10 at $\alpha=0.05$.

Assumptions: The observations are independent and the population is approximately normal.
Reference: One-sample $t=(\bar{x}-\mu_0)/(s/\sqrt{n})$, $df=n-1$. t reference: $P(T_5 \geq 8.818163074)=0.0001557151$; two-sided $p=2(0.0001557151)=0.0003114302$. Critical $|t|=2.570581836$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

7. Independent samples give group 1: $n=7$, $\text{mean}=20$, $s=3$; group 2: $n=10$, $\text{mean}=15$, $s=3$. Use the equal-variance pooled test of $H_0: \mu_1 - \mu_2 = 0$ at $\alpha=0.1$.

Assumptions: Samples are independent and each population is approximately normal. Population variances are assumed equal.
Reference: Pool the variance and use $df=n_1+n_2-2$. t reference: $P(T_{15} \geq 3.381997708) = 0.0020533544$; two-sided $p=2(0.0020533544) = 0.0041067088$. Critical $|t|=1.753050356$.

One-Sample, Paired, Pooled, and Welch t Tests

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

8. Independent samples give group 1: $n=8$, mean=14, $s=3.5$; group 2: $n=11$, mean=15, $s=7$. Use the unequal-variance Welch test of $H_0: \mu_1 - \mu_2 = 0$ at $\alpha=0.01$.

Assumptions: Samples are independent and each population is approximately normal. Equal variances are not assumed.
Reference: Use $SE = \sqrt{s_1^2/n_1 + s_2^2/n_2}$ and Welch-Satterthwaite df without rounding. t reference: $P(T_{15.448794462} > 0.408732399) = 0.3441729238$; two-sided $p = 2(0.3441729238) = 0.6883458476$. Critical $|t| = 2.934610110$.

One-Sample, Paired, Pooled, and Welch t Tests

Worked Answer Key

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

1. Independent samples give group 1: n=6, mean=14, s=2; group 2: n=9, mean=15, s=3. Use the equal-variance pooled test of H0: $\mu_1 - \mu_2 = 0$ at $\alpha = 0.1$.

Assumptions: Samples are independent and each population is approximately normal. Population variances are assumed equal.

Reference: Pool the variance and use $df = n_1 + n_2 - 2$. t reference: $P(T_{13} > 0.713229032) = 0.2441552908$; two-sided $p = 2(0.2441552908) = 0.4883105816$. Critical $|t| = 1.770933396$.

Answer: $t = -0.7132$, $df = 13$, $p = 0.4883$; Fail to reject H0 at $\alpha = 0.1$; the data do not establish that H0 is true.

- 1. $s_p^2 = 7.0769$; $df = 13$.
- 2. $t = -0.713229032$; use the printed survival value to obtain $p = 0.4883105816$.
- 3. Fail to reject H0 at $\alpha = 0.1$; the data do not establish that H0 is true.

One-Sample, Paired, Pooled, and Welch t Tests
Worked Answer Key

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

2. Matched before values are 13,16,15,19,17; after values are 14,18,14,22,20. Test whether the mean paired change differs from 0 at alpha=0.01.

Assumptions: Pairs are independent of other pairs; paired differences are approximately normal.
Reference: Use $d = \text{after} - \text{before}$, $t = \bar{d} / (s_d / \sqrt{n})$, $df = n - 1$. t reference: $P(T_4 \geq 2.138089935) = 0.0496503416$; two-sided $p = 2(0.0496503416) = 0.0993006832$. Critical $|t| = 4.604094871$.

Answer: $t = 2.1381$, $df = 4$, $p = 0.0993$; Fail to reject H_0 at $\alpha = 0.01$; the data do not establish that H_0 is true.

- 1. Differences: 1,2,-1,3,3; $\bar{d} = 1.6$, $s_d = 1.6733$.
- 2. $t = 2.138089935$, $df = 4$; use the printed tail to obtain $p = 0.0993006832$.
- 3. Fail to reject H_0 at $\alpha = 0.01$; the data do not establish that H_0 is true.

One-Sample, Paired, Pooled, and Welch t Tests

Worked Answer Key

Identify the design, statistic, degrees of freedom, assumptions, and conclusion. Keep Welch degrees of freedom unrounded during probability calculations.

3. A sample has $n=8$, $\bar{x}=16$, and $s=3$. Test $H_0: \mu=12$ against $H_A: \mu$ differs from 12 at $\alpha=0.05$.

Assumptions: The observations are independent and the population is approximately normal.

Reference: One-sample $t=(\bar{x}-\mu_0)/(s/\sqrt{n})$, $df=n-1$. t reference: $P(T_7 \geq 3.771236166)=0.0034858275$; two-sided $p=2(0.0034858275)=0.0069716550$. Critical $|t|=2.364624252$.

Answer: $t=3.7712$, $df=7$, $p=0.007$; Reject H_0 at $\alpha=0.05$; the data support H_A .

- 1. $SE=1.0607$; $t=3.771236166$.
- 2. Use the printed tail at $df=7$ to obtain $p=0.0069716550$.
- 3. Reject H_0 at $\alpha=0.05$; the data support H_A .

One-Sample, Paired, Pooled, and Welch t Tests
Worked Answer Key

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4. Independent samples give group 1: $n=9$, $\text{mean}=17$, $s=3.5$; group 2: $n=7$, $\text{mean}=15$, $s=7$. Use the unequal-variance Welch test of $H_0: \mu_1-\mu_2=0$ at $\alpha=0.1$.

Assumptions: Samples are independent and each population is approximately normal. Equal variances are not assumed.

Reference: Use $SE=\sqrt{(s_1^2/n_1+s_2^2/n_2)}$ and Welch-Satterthwaite df without rounding. t reference: $P(T_{8.324141812}\geq 0.691668501)=0.2539874797$; two-sided $p=2(0.2539874797)=0.5079749594$. Critical $|t|=1.850204075$.

Answer: $t=0.6917$, $df=8.3241$, $p=0.508$; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

- 1. Welch $df=8.324141812$ (retain this noninteger value).
- 2. $t=0.691668501$; use the printed survival value to obtain $p=0.5079749594$.
- 3. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

One-Sample, Paired, Pooled, and Welch t Tests

Worked Answer Key

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5. Matched before values are 16,19,18,22,20; after values are 17,21,17,25,22. Test whether the mean paired change differs from 0 at alpha=0.01.

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Reference: Use d=after-before, $t = \frac{\bar{d}}{s_d/\sqrt{n}}$, df=n-1. t reference: $P(T_4 \geq 2.064187386) = 0.0539694112$; two-sided $p = 2(0.0539694112) = 0.1079388224$. Critical $|t| = 4.604094871$.

Answer: t=2.0642, df=4, p=0.1079; Fail to reject H0 at alpha=0.01; the data do not establish that H0 is true.

- 1. Differences: 1,2,-1,3,2; d-bar=1.4, s_d=1.5166.
- 2. t=2.064187386, df=4; use the printed tail to obtain p=0.1079388224.
- 3. Fail to reject H0 at alpha=0.01; the data do not establish that H0 is true.

One-Sample, Paired, Pooled, and Welch t Tests

Worked Answer Key

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6. A sample has $n=6$, $\bar{x}=19$, and $s=2.5$. Test $H_0: \mu=10$ against $H_A: \mu \text{ differs from } 10$ at $\alpha=0.05$.

Assumptions: The observations are independent and the population is approximately normal.

Reference: One-sample $t=(\bar{x}-\mu_0)/(s/\sqrt{n})$, $df=n-1$. t reference: $P(T_5 \geq 8.818163074)=0.0001557151$; two-sided $p=2(0.0001557151)=0.0003114302$. Critical $|t|=2.570581836$.

Answer: $t=8.8182$, $df=5$, $p=0.0003$; Reject H_0 at $\alpha=0.05$; the data support H_A .

- 1. $SE=1.0206$; $t=8.818163074$.
- 2. Use the printed tail at $df=5$ to obtain $p=0.0003114302$.
- 3. Reject H_0 at $\alpha=0.05$; the data support H_A .

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Worked Answer Key

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7. Independent samples give group 1: $n=7$, $\text{mean}=20$, $s=3$; group 2: $n=10$, $\text{mean}=15$, $s=3$. Use the equal-variance pooled test of $H_0: \mu_1-\mu_2=0$ at $\alpha=0.1$.

Assumptions: Samples are independent and each population is approximately normal. Population variances are assumed equal.

Reference: Pool the variance and use $df=n_1+n_2-2$. t reference: $P(T_{15} \geq 3.381997708) = 0.0020533544$; two-sided $p=2(0.0020533544) = 0.0041067088$. Critical $|t|=1.753050356$.

Answer: $t=3.3820$, $df=15$, $p=0.0041$; Reject H_0 at $\alpha=0.1$; the data support H_A .

- 1. $s_p^2=9$; $df=15$.
- 2. $t=3.381997708$; use the printed survival value to obtain $p=0.0041067088$.
- 3. Reject H_0 at $\alpha=0.1$; the data support H_A .

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8. Independent samples give group 1: $n=8$, $\text{mean}=14$, $s=3.5$; group 2: $n=11$, $\text{mean}=15$, $s=7$. Use the unequal-variance Welch test of $H_0: \mu_1 - \mu_2 = 0$ at $\alpha=0.01$.

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Answer: $t = -0.4087$, $df = 15.4488$, $p = 0.6883$; Fail to reject H_0 at $\alpha = 0.01$; the data do not establish that H_0 is true.

- 1. Welch $df = 15.448794462$ (retain this noninteger value).
- 2. $t = -0.408732399$; use the printed survival value to obtain $p = 0.6883458476$.
- 3. Fail to reject H_0 at $\alpha = 0.01$; the data do not establish that H_0 is true.