

Prior Choice and Sensitivity

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

1. Reliable background evidence has an average success rate 0.2 and strength about 5. Choose the most defensible prior from Beta(1,4), Beta(4,1), Beta(1,4) and explain sensitivity to limited new data.

Given: Prior choice should reflect both center and strength of genuine background evidence.

2. Apply identical data (5 successes, 3 failures) to Beta(1,1), Beta(2,4), Beta(4,2). Compare posterior means and explain sensitivity.

Given: Every prior sees the same Bernoulli data.

| Prior     | Posterior | Posterior mean |
|-----------|-----------|----------------|
| Beta(1,1) | Beta(6,4) | 0.6000         |
| Beta(2,4) | Beta(7,7) | 0.5000         |
| Beta(4,2) | Beta(9,5) | 0.6429         |

Prior Choice and Sensitivity

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

3. Compare weak Beta(1,1) and strong Beta(8,8) priors after a small batch (2 successes, 1 failures) and a larger batch (4 successes, 2 failures) with the same success proportion 2/3.

Given: Both priors see each batch, and the larger batch preserves the small batch's success-to-failure ratio.

| Prior     | Batch     | Posterior   | Mean   |
|-----------|-----------|-------------|--------|
| Beta(1,1) | small 2:1 | Beta(3,2)   | 0.6000 |
| Beta(1,1) | large 4:2 | Beta(5,3)   | 0.6250 |
| Beta(8,8) | small 2:1 | Beta(10,9)  | 0.5263 |
| Beta(8,8) | large 4:2 | Beta(12,10) | 0.5455 |

4. A discrete prior over H1,H2,H3 is (0.00, 0.60, 0.40); likelihoods are (0.70, 0.60, 0.25). Find the posterior and explain whether strong evidence can revive H1.

Given: Ordinary updating cannot add mass where prior mass is exactly zero.

Prior Choice and Sensitivity

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

5. Compare weak Beta(1,1) and strong Beta(10,10) priors after a small batch (3 successes, 1 failures) and a larger batch (6 successes, 2 failures) with the same success proportion 3/4.

Given: Both priors see each batch, and the larger batch preserves the small batch's success-to-failure ratio.

| Prior       | Batch     | Posterior   | Mean   |
|-------------|-----------|-------------|--------|
| Beta(1,1)   | small 3:1 | Beta(4,2)   | 0.6667 |
| Beta(1,1)   | large 6:2 | Beta(7,3)   | 0.7000 |
| Beta(10,10) | small 3:1 | Beta(13,11) | 0.5417 |
| Beta(10,10) | large 6:2 | Beta(16,12) | 0.5714 |

6. A discrete prior over H1,H2,H3 is (0.00, 0.80, 0.20); likelihoods are (0.90, 0.60, 0.15). Find the posterior and explain whether strong evidence can revive H1.

Given: Ordinary updating cannot add mass where prior mass is exactly zero.

Prior Choice and Sensitivity

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

7. Reliable background evidence has an average success rate 0.2 and strength about 15. Choose the most defensible prior from Beta(3,12), Beta(4,1), Beta(1,4) and explain sensitivity to limited new data.

Given: Prior choice should reflect both center and strength of genuine background evidence.

8. Apply identical data (6 successes, 1 failures) to Beta(1,1), Beta(2,4), Beta(4,2). Compare posterior means and explain sensitivity.

Given: Every prior sees the same Bernoulli data.

| Prior     | Posterior  | Posterior mean |
|-----------|------------|----------------|
| Beta(1,1) | Beta(7,2)  | 0.7778         |
| Beta(2,4) | Beta(8,5)  | 0.6154         |
| Beta(4,2) | Beta(10,3) | 0.7692         |

Prior Choice and Sensitivity

Worked Answer Key

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

1. Reliable background evidence has an average success rate 0.2 and strength about 5. Choose the most defensible prior from Beta(1,4), Beta(4,1), Beta(1,4) and explain sensitivity to limited new data.

Answer: Beta(1,4)

- 1. Choose Beta(1,4): its mean is 0.2 and strength is 5.
- 2. A prior with the same mean but lower strength would move more after limited data.
- 3. Report alternative-prior results when background evidence is uncertain.

2. Apply identical data (5 successes, 3 failures) to Beta(1,1), Beta(2,4), Beta(4,2). Compare posterior means and explain sensitivity.

| Prior     | Posterior | Posterior mean |
|-----------|-----------|----------------|
| Beta(1,1) | Beta(6,4) | 0.6000         |
| Beta(2,4) | Beta(7,7) | 0.5000         |
| Beta(4,2) | Beta(9,5) | 0.6429         |

Answer: Beta(6,4) mean 0.6000; Beta(7,7) mean 0.5000; Beta(9,5) mean 0.6429

- 1. Add (5,3) to every prior shape pair.
- 2. Posterior means are 0.6000, 0.5000, 0.6429.
- 3. Differences reveal sensitivity to the prior under this bounded sample.

# Math Sheet

Name \_\_\_\_\_ Score \_\_\_\_\_ Teacher \_\_\_\_\_ Date \_\_\_\_\_

## Prior Choice and Sensitivity

### Worked Answer Key

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

3. Compare weak Beta(1,1) and strong Beta(8,8) priors after a small batch (2 successes, 1 failures) and a larger batch (4 successes, 2 failures) with the same success proportion 2/3.

| Prior     | Batch     | Posterior   | Mean   |
|-----------|-----------|-------------|--------|
| Beta(1,1) | small 2:1 | Beta(3,2)   | 0.6000 |
| Beta(1,1) | large 4:2 | Beta(5,3)   | 0.6250 |
| Beta(8,8) | small 2:1 | Beta(10,9)  | 0.5263 |
| Beta(8,8) | large 4:2 | Beta(12,10) | 0.5455 |

Answer: Small-batch posterior means: 0.6000 and 0.5263; large-batch means: 0.6250 and 0.5455

1. The matched data proportion is  $2/3=0.6667$  in both batches.
2. The weak prior posterior moves closer to the data proportion than the strong prior after the small batch.
3. With the larger matched batch, both posterior means move strictly toward 0.6667; the stronger prior remains more resistant to the data.

4. A discrete prior over  $H_1, H_2, H_3$  is (0.00, 0.60, 0.40); likelihoods are (0.70, 0.60, 0.25). Find the posterior and explain whether strong evidence can revive  $H_1$ .

Answer: Posterior (0.0000, 0.7826, 0.2174);  $H_1$  stays zero

1.  $H_1$  numerator=0 times its likelihood=0.
2. Normalize the other two numerators.
3. A zero prior is rigid: no finite likelihood ratio revives  $H_1$ .

Prior Choice and Sensitivity

Worked Answer Key

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

5. Compare weak Beta(1,1) and strong Beta(10,10) priors after a small batch (3 successes, 1 failures) and a larger batch (6 successes, 2 failures) with the same success proportion 3/4.

| Prior       | Batch     | Posterior   | Mean   |
|-------------|-----------|-------------|--------|
| Beta(1,1)   | small 3:1 | Beta(4,2)   | 0.6667 |
| Beta(1,1)   | large 6:2 | Beta(7,3)   | 0.7000 |
| Beta(10,10) | small 3:1 | Beta(13,11) | 0.5417 |
| Beta(10,10) | large 6:2 | Beta(16,12) | 0.5714 |

Answer: Small-batch posterior means: 0.6667 and 0.5417; large-batch means: 0.7000 and 0.5714

- The matched data proportion is 3/4=0.7500 in both batches.
- The weak prior posterior moves closer to the data proportion than the strong prior after the small batch.
- With the larger matched batch, both posterior means move strictly toward 0.7500; the stronger prior remains more resistant to the data.

6. A discrete prior over H1,H2,H3 is (0.00, 0.80, 0.20); likelihoods are (0.90, 0.60, 0.15). Find the posterior and explain whether strong evidence can revive H1.

Answer: Posterior (0.0000, 0.9412, 0.0588); H1 stays zero

- H1 numerator=0 times its likelihood=0.
- Normalize the other two numerators.
- A zero prior is rigid: no finite likelihood ratio revives H1.

Prior Choice and Sensitivity

Worked Answer Key

Apply identical data to every candidate prior. Compare posterior means and strengths, note when a strong prior resists limited data, and remember that a hypothesis assigned prior probability zero remains at zero after ordinary updating.

7. Reliable background evidence has an average success rate 0.2 and strength about 15. Choose the most defensible prior from Beta(3,12), Beta(4,1), Beta(1,4) and explain sensitivity to limited new data.

Answer: Beta(3,12)

- 1. Choose Beta(3,12): its mean is 0.2 and strength is 15.
- 2. A prior with the same mean but lower strength would move more after limited data.
- 3. Report alternative-prior results when background evidence is uncertain.

8. Apply identical data (6 successes, 1 failures) to Beta(1,1), Beta(2,4), Beta(4,2). Compare posterior means and explain sensitivity.

| Prior     | Posterior  | Posterior mean |
|-----------|------------|----------------|
| Beta(1,1) | Beta(7,2)  | 0.7778         |
| Beta(2,4) | Beta(8,5)  | 0.6154         |
| Beta(4,2) | Beta(10,3) | 0.7692         |

Answer: Beta(7,2) mean 0.7778; Beta(8,5) mean 0.6154; Beta(10,3) mean 0.7692

- 1. Add (6,1) to every prior shape pair.
- 2. Posterior means are 0.7778, 0.6154, 0.7692.
- 3. Differences reveal sensitivity to the prior under this bounded sample.