

One-Way ANOVA

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

1. Groups are A=[10,11,12]; B=[11,12,13]; C=[12,13,14]. Compute each group mean, the grand mean, SS_between, and SS_within.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

One-Way ANOVA

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

2. Groups are A=[12,13,14,15]; B=[14,15,16,17]; C=[16,17,18,19]; D=[19,20,21,22]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at $\alpha=0.1$.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{3,12} \geq 21.40000000) = 0.0000416005$; critical $F = 2.605524921$.

Name the conditioning distribution, show the statistic and df, then state a contextual decision.

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3. Groups are A=[14,15,16,17,18]; B=[17,18,19,20,21]; C=[20,21,22,23,24]. Given MS_between=45 and MS_within=2.5, perform the omnibus F test of equal population means at alpha=0.01; report F, its degrees of freedom, the p-value, and a contextual interpretation.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: F=MS_between/MS_within with df=(2,12). F reference: P(F_2,12>=18.000000000)=0.0002441406; critical F=6.926608140.

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4. Groups are A=[16,17,18]; B=[17,18,19]; C=[18,19,20]; D=[20,21,22]. Compute each group mean, the grand mean, SS_between, and SS_within.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.

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5. Groups are A=[18,19,20,21]; B=[20,21,22,23]; C=[22,23,24,25]. Given MS_between=16 and MS_within=1.6667, perform the omnibus F test of equal population means at alpha=0.1; report F, its degrees of freedom, the p-value, and a contextual interpretation.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: F=MS_between/MS_within with df=(2,9). F reference: P(F_2,9>=9.600000000)=0.0058609809; critical F=3.006452417.

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6. Groups are A=[20,21,22,23,24]; B=[23,24,25,26,27]; C=[26,27,28,29,30]; D=[30,31,32,33,34]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at alpha=0.01.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{3,16} \geq 36.500000000) = 0.0000002194$; critical $F = 5.292214046$.

One-Way ANOVA

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

7. Groups are A=[22,23,24]; B=[23,24,25]; C=[24,25,26]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at alpha=0.05.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{2,6} \geq 3.000000000) = 0.1250000000$; critical $F = 5.143252850$.

One-Way ANOVA

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

8. Groups are A=[24,25,26,27]; B=[26,27,28,29]; C=[28,29,30,31]; D=[31,32,33,34]. Compute each group mean, the grand mean, SS_between, and SS_within.
Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.

One-Way ANOVA
Worked Answer Key

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

1. Groups are A=[10,11,12]; B=[11,12,13]; C=[12,13,14]. Compute each group mean, the grand mean, SS_between, and SS_within.
- Assumptions: Samples are independent; each population is approximately normal with a common variance.
- Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.
- Answer: Group means=11, 12, 13; grand mean=12. SS_between=6 and SS_within=6; SS_total=12.**
- 1. Group means=11, 12, 13; grand mean=12.
 - 2. $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2 = 6$.
 - 3. $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2 = 6$; check $SS_{\text{total}} = 12$.

One-Way ANOVA
Worked Answer Key

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

2. Groups are A=[12,13,14,15]; B=[14,15,16,17]; C=[16,17,18,19]; D=[19,20,21,22]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at alpha=0.1.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{3,12} \geq 21.400000000) = 0.0000416005$; critical $F = 2.605524921$.

Answer: Between: SS=107, df=3, MS=35.6667. Within: SS=20, df=12, MS=1.6667. Total: SS=127, df=15. F=21.4000, p=0; Reject H0 at alpha=0.1; the data support HA. At least one population mean differs; this does not show that every pair differs.

- 1. Group means=13.5, 15.5, 17.5, 20.5; grand mean=16.75.
- 2. Between: SS=107, df=3, MS=35.6667. Within: SS=20, df=12, MS=1.6667. Total: SS=127, df=15.
- 3. F=21.400000000; use the printed F-tail $p = 0.0000416005$.
- 4. Reject H0 at alpha=0.1; the data support HA. At least one population mean differs; this does not show that every pair differs.

One-Way ANOVA

Worked Answer Key

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

3. Groups are A=[14,15,16,17,18]; B=[17,18,19,20,21]; C=[20,21,22,23,24]. Given MS_between=45 and MS_within=2.5, perform the omnibus F test of equal population means at $\alpha=0.01$; report F, its degrees of freedom, the p-value, and a contextual interpretation.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: $F = MS_{\text{between}} / MS_{\text{within}}$ with $df=(2,12)$. F reference: $P(F_{2,12} \geq 18.000000000) = 0.0002441406$; critical $F=6.926608140$.

Answer: $F=18.0000$, $df=(2,12)$, $p=0.0002$; Reject H_0 at $\alpha=0.01$; the data support H_A . At least one population mean differs; this does not show that every pair differs.

1. $F=45/2.5=18.000000000$.
2. Use the printed F-tail $p=0.0002441406$ with $\alpha=0.01$.
3. Reject H_0 at $\alpha=0.01$; the data support H_A . At least one population mean differs; this does not show that every pair differs.

One-Way ANOVA
Worked Answer Key

Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

4. Groups are A=[16,17,18]; B=[17,18,19]; C=[18,19,20]; D=[20,21,22]. Compute each group mean, the grand mean, SS_between, and SS_within.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.

Answer: Group means=17, 18, 19, 21; grand mean=18.75. SS_between=26.25 and SS_within=8; SS_total=34.25.

- 1. Group means=17, 18, 19, 21; grand mean=18.75.
- 2. $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2 = 26.25$.
- 3. $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2 = 8$; check $SS_{\text{total}} = 34.25$.

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Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

5. Groups are A=[18,19,20,21]; B=[20,21,22,23]; C=[22,23,24,25]. Given MS_between=16 and MS_within=1.6667, perform the omnibus F test of equal population means at $\alpha=0.1$; report F, its degrees of freedom, the p-value, and a contextual interpretation.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: $F = MS_{\text{between}} / MS_{\text{within}}$ with $df=(2,9)$. F reference: $P(F_{2,9} > 9.600000000) = 0.0058609809$; critical $F = 3.006452417$.

Answer: $F=9.6000$, $df=(2,9)$, $p=0.0059$; Reject H_0 at $\alpha=0.1$; the data support H_A . At least one population mean differs; this does not show that every pair differs.

1. $F=16/1.6667=9.600000000$.
2. Use the printed F-tail $p=0.0058609809$ with $\alpha=0.1$.
3. Reject H_0 at $\alpha=0.1$; the data support H_A . At least one population mean differs; this does not show that every pair differs.

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Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

6. Groups are A=[20,21,22,23,24]; B=[23,24,25,26,27]; C=[26,27,28,29,30]; D=[30,31,32,33,34]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at alpha=0.01.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{3,16} \geq 36.500000000) = 0.0000002194$; critical $F = 5.292214046$.

Answer: Between: SS=273.75, df=3, MS=91.25. Within: SS=40, df=16, MS=2.5. Total: SS=313.75, df=19. F=36.5000, p=0; Reject H0 at alpha=0.01; the data support HA. At least one population mean differs; this does not show that every pair differs.

- 1. Group means=22, 25, 28, 32; grand mean=26.75.
- 2. Between: SS=273.75, df=3, MS=91.25. Within: SS=40, df=16, MS=2.5. Total: SS=313.75, df=19.
- 3. F=36.500000000; use the printed F-tail p=0.0000002194.
- 4. Reject H0 at alpha=0.01; the data support HA. At least one population mean differs; this does not show that every pair differs.

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Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

7. Groups are A=[22,23,24]; B=[23,24,25]; C=[24,25,26]. Complete the between, within, and total rows of the one-way ANOVA table, then test equal population means at alpha=0.05.

Assumptions: Samples are independent; each population is approximately normal with a common variance.
Reference: Table columns are source, SS, df, MS, and F (F only in the between row). F reference: $P(F_{2,6} > 3.000000000) = 0.1250000000$; critical $F = 5.143252850$.

Answer: Between: SS=6, df=2, MS=3. Within: SS=6, df=6, MS=1. Total: SS=12, df=8. $F = 3.0000$, $p = 0.125$; Fail to reject H_0 at $\alpha = 0.05$; the data do not establish that H_0 is true. No mean difference is established by this omnibus test.

- 1. Group means=23, 24, 25; grand mean=24.
- 2. Between: SS=6, df=2, MS=3. Within: SS=6, df=6, MS=1. Total: SS=12, df=8.
- 3. $F = 3.000000000$; use the printed F-tail $p = 0.1250000000$.
- 4. Fail to reject H_0 at $\alpha = 0.05$; the data do not establish that H_0 is true. No mean difference is established by this omnibus test.

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Show group and grand means, between and within sums of squares, the complete ANOVA table, and the omnibus conclusion.

8. Groups are A=[24,25,26,27]; B=[26,27,28,29]; C=[28,29,30,31]; D=[31,32,33,34]. Compute each group mean, the grand mean, SS_between, and SS_within.

Assumptions: Samples are independent; each population is approximately normal with a common variance.

Reference: Use $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2$ and $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2$.

Answer: Group means=25.5, 27.5, 29.5, 32.5; grand mean=28.75. SS_between=107 and SS_within=20; SS_total=127.

- 1. Group means=25.5, 27.5, 29.5, 32.5; grand mean=28.75.
- 2. $SS_{\text{between}} = \sum n_j(\bar{x}_j - \bar{x})^2 = 107$.
- 3. $SS_{\text{within}} = \sum \sum (x_{ij} - \bar{x}_j)^2 = 20$; check $SS_{\text{total}} = 127$.