

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

- 1. A random sample from a normal population has $n=7$ and sample variance $s^2=9$. Construct a 90% confidence interval for population variance σ^2 .**

Reference: With $df=6$, lower-tail chi-square quantiles $q_L=1.6353829$ and $q_U=12.5915872$, use $((df)s^2/q_U, (df)s^2/q_L)$. Denominators reverse.

- 2. A normal population has known $\sigma=5$. A sample of $n=8$ has mean 43. Construct a 95% z confidence interval for μ .**

Reference: Known population sigma: $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.

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Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

- 3. For known $\sigma=6$, $n=9$, and $\bar{x}=46$, invert the two-sided level 0.05 z test of $H_0: \mu=\mu_0$. Give all μ_0 values not rejected.**

Reference: The two-sided z test keeps H_0 when $|(\bar{x}-\mu_0)/(\sigma/\sqrt{n})| \leq z_{0.975}$, with $z_{0.975}=1.9599639$. Solving gives the matching 95% confidence interval.

- 4. Ten repeated-sample intervals for the same parameter are (66,70), (66,72), (75,79), (65,71), (67,71), (67,73), (66,70), (73,79), (68,72), (65,71). The true value is 69. Find observed coverage and interpret it.**

Reference: An interval covers when lower $\leq$ true value $\leq$ upper. Long-run confidence concerns the procedure across repeated samples, not a probability assigned to this fixed parameter.

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Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

- 5. A random sample from a normal population has $n=11$, $\bar{x}=52$, and sample SD $s=7$; population variance is unknown. Construct a 95% t confidence interval for μ .**

Reference: Unknown population sigma with normal data: $\bar{x} \pm t_{(1-\alpha/2, n-1)}s/\sqrt{n}$. Lower-tail reference $t_{(1-\alpha/2, 10)}=2.2281389$.

- 6. A student proposes $\bar{x} \pm 1.5s/\sqrt{n}$ as a data-based interval, where μ is the unknown population mean. Is it an interval statistic computable before μ is known?**

Reference: An interval statistic may use observations and known constants, but its endpoints cannot contain the unknown target parameter.

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Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

7. Evaluate this statement: "There is a 86% probability that this fixed μ lies in the computed frequentist interval."

Reference: A frequentist confidence level is long-run procedure coverage. A Bayesian credible probability is conditional on a stated prior and model.

8. A normal population has known $\sigma=11$. A sample of $n=14$ has mean 61. Construct a 95% z confidence interval for μ .

Reference: Known population sigma: $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.

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Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

1. A random sample from a normal population has $n=7$ and sample variance $s^2=9$. Construct a 90% confidence interval for population variance σ^2 .

Reference: With $df=6$, lower-tail chi-square quantiles $q_L=1.6353829$ and $q_U=12.5915872$, use $((df)s^2/q_U, (df)s^2/q_L)$. Denominators reverse.

Answer: (4.289, 33.02)

1. $df=6$; printed $q_L=1.6353829$ and $q_U=12.5915872$.
2. Lower= $6(9)/12.5915872=4.289$; upper= $6(9)/1.6353829=33.02$.

2. A normal population has known $\sigma=5$. A sample of $n=8$ has mean 43. Construct a 95% z confidence interval for μ .

Reference: Known population σ : $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.

Answer: (39.535, 46.465)

1. Printed $z_{(1-\alpha/2)}=1.9599639$. Margin= $1.9599639(5)/\sqrt{8}=3.465$.
2. Interval= $43 \pm 3.465=(39.535, 46.465)$.

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Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

3. For known $\sigma=6$, $n=9$, and $\bar{x}=46$, invert the two-sided level 0.05 z test of $H_0: \mu=\mu_0$. Give all μ_0 values not rejected.

Reference: The two-sided z test keeps H_0 when $|(\bar{x}-\mu_0)/(\sigma/\sqrt{n})| \leq z_{0.975}$, with $z_{0.975}=1.9599639$. Solving gives the matching 95% confidence interval.

Answer: μ_0 in [42.08, 49.92]

1. Printed $z_{0.975}=1.9599639$; solve $|46-\mu_0| \leq 1.9599639(6)/\sqrt{9}$.
2. The nonrejected values are [42.08, 49.92], including both endpoints under the stated $\leq$ rule.

4. Ten repeated-sample intervals for the same parameter are (66,70), (66,72), (75,79), (65,71), (67,71), (67,73), (66,70), (73,79), (68,72), (65,71). The true value is 69. Find observed coverage and interpret it.

Reference: An interval covers when lower $\leq$ true value $\leq$ upper. Long-run confidence concerns the procedure across repeated samples, not a probability assigned to this fixed parameter.

Answer: 8/10 = 80%

1. Count intervals containing 69: 8 of 10.
2. This small run has 80% observed coverage; it can differ substantially from the nominal level.

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Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

5. A random sample from a normal population has $n=11$, $\bar{x}=52$, and sample SD $s=7$; population variance is unknown. Construct a 95% t confidence interval for μ .

Reference: Unknown population sigma with normal data: $\bar{x} \pm t_{(1-\alpha/2, n-1)}s/\sqrt{n}$. Lower-tail reference $t_{(1-\alpha/2, 10)}=2.2281389$.

Answer: (47.297, 56.703)

1. $df=10$; printed $t_{(1-\alpha/2, df)}=2.2281389$.
2. Margin= $2.2281389(7)/\sqrt{11}=4.703$; interval= $(47.297, 56.703)$.

6. A student proposes $\bar{x} \pm 1.5s/\sqrt{n}$ as a data-based interval, where μ is the unknown population mean. Is it an interval statistic computable before μ is known?

Reference: An interval statistic may use observations and known constants, but its endpoints cannot contain the unknown target parameter.

Answer: Yes; its endpoints use sample statistics only

1. $\bar{x}$, s , and n are computable from the sample; no unknown population parameter appears.

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

7. Evaluate this statement: "There is a 86% probability that this fixed μ lies in the computed frequentist interval."

Reference: A frequentist confidence level is long-run procedure coverage. A Bayesian credible probability is conditional on a stated prior and model.

Answer: Incorrect as a frequentist interpretation

1. After data are observed, the frequentist endpoints and fixed parameter are not random under the repeated-sampling interpretation.

8. A normal population has known $\sigma=11$. A sample of $n=14$ has mean 61. Construct a 95% z confidence interval for μ .

Reference: Known population σ : $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.

Answer: (55.238, 66.762)

1. Printed $z_{(1-\alpha/2)}=1.9599639$. Margin= $1.9599639(11)/\sqrt{14}=5.762$.
2. Interval= $61 \pm 5.762=(55.238, 66.762)$.