

Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

1. A random sample from a normal population has $n=7$ and sample variance $s^2=9$. Construct a 90% confidence interval for population variance σ^2 .

Reference: With $df=6$, lower-tail chi-square quantiles $q_L=1.6353829$ and $q_U=12.5915872$, use $((df)s^2/q_U, (df)s^2/q_L)$. Denominators reverse.

2. A normal population has known $\sigma=5$. A sample of $n=8$ has mean 43. Construct a 95% z confidence interval for μ .

Reference: Known population sigma: $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.

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3.

For known $\sigma=6$, $n=9$, and $\bar{x}=46$, invert the two-sided level 0.05 z test of $H_0: \mu=\mu_0$. Give all μ_0 values not rejected.

Reference: The two-sided z test keeps H_0 when $|(\bar{x}-\mu_0)/(\sigma/\sqrt{n})| \leq z_{0.975}$, with $z_{0.975}=1.9599639$. Solving gives the matching 95% confidence interval.

4.

Ten repeated-sample intervals for the same parameter are (66,70), (66,72), (75,79), (65,71), (67,71), (67,73), (66,70), (73,79), (68,72), (65,71). The true value is 69. Find observed coverage and interpret it.

Reference: An interval covers when lower $\leq$ true value $\leq$ upper. Long-run confidence concerns the procedure across repeated samples, not a probability assigned to this fixed parameter.

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5. A random sample from a normal population has $n=11$, $\bar{x}=52$, and sample SD $s=7$; population variance is unknown. Construct a 95% t confidence interval for μ .

Reference: Unknown population sigma with normal data: $\bar{x} \pm t_{(1-\alpha/2, n-1)}s/\sqrt{n}$. Lower-tail reference $t_{(1-\alpha/2, 10)}=2.2281389$.

6. A student proposes $\bar{x} \pm 1.5s/\sqrt{n}$ as a data-based interval, where μ is the unknown population mean. Is it an interval statistic computable before μ is known?

Reference: An interval statistic may use observations and known constants, but its endpoints cannot contain the unknown target parameter.

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7. Evaluate this statement: “There is a 86% probability that this fixed mu lies in the computed frequentist interval.”

Reference: A frequentist confidence level is long-run procedure coverage. A Bayesian credible probability is conditional on a stated prior and model.

8. A normal population has known sigma=11. A sample of n=14 has mean 61. Construct a 95% z confidence interval for mu.

Reference: Known population sigma: $\bar{x} \pm z_{(1-\alpha/2)} \sigma/\sqrt{n}$. Lower-tail reference $z_{(1-\alpha/2)}=1.9599639$.