Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

1. **A random sample from a normal population has n=7 and sample variance s^2=9. Construct a 90% confidence interval for population variance sigma^2.**

Reference: With df=6, lower-tail chi-square quantiles q_L=1.6353829 and q_U=12.5915872, use ((df)s^2/q_U, (df)s^2/q_L). Denominators reverse.

2. **A normal population has known sigma=5. A sample of n=8 has mean 43. Construct a 95% z confidence interval for mu.**

Reference: Known population sigma: x-bar +/- z_(1-alpha/2) sigma/sqrt(n). Lower-tail reference z_(1-alpha/2)=1.9599639.

State assumptions and the quantile convention; round only final endpoints.

Name _____ Score _____ Teacher _____ Date _____

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Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

3. **For known sigma=6, n=9, and x-bar=46, invert the two-sided level 0.05 z test of H0: mu=mu0. Give all mu0 values not rejected.**

Reference: The two-sided z test keeps H0 when |(x-bar-mu0)/(sigma/sqrt(n))|<=z_0.975, with z_0.975=1.9599639. Solving gives the matching 95% confidence interval.

4. **Ten repeated-sample intervals for the same parameter are (66,70), (66,72), (75,79), (65,71), (67,71), (67,73), (66,70), (73,79), (68,72), (65,71). The true value is 69. Find observed coverage and interpret it.**

Reference: An interval covers when lower <= true value <= upper. Long-run confidence concerns the procedure across repeated samples, not a probability assigned to this fixed parameter.

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

5. **A random sample from a normal population has n=11, x-bar=52, and sample SD s=7; population variance is unknown. Construct a 95% t confidence interval for mu.**

Reference: Unknown population sigma with normal data: x-bar +/- t_(1-alpha/2,n-1)s/sqrt(n). Lower-tail reference t_(1-alpha/2,10)=2.2281389.

6. **A student proposes x-bar +/- 1.5s/sqrt(n) as a data-based interval, where mu is the unknown population mean. Is it an interval statistic computable before mu is known?**

Reference: An interval statistic may use observations and known constants, but its endpoints cannot contain the unknown target parameter.

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

7. **Evaluate this statement: “There is a 86% probability that this fixed mu lies in the computed frequentist interval.”**

Reference: A frequentist confidence level is long-run procedure coverage. A Bayesian credible probability is conditional on a stated prior and model.

8. **A normal population has known sigma=11. A sample of n=14 has mean 61. Construct a 95% z confidence interval for mu.**

Reference: Known population sigma: x-bar +/- z_(1-alpha/2) sigma/sqrt(n). Lower-tail reference z_(1-alpha/2)=1.9599639.

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

1. **A random sample from a normal population has n=7 and sample variance s^2=9. Construct a 90% confidence interval for population variance sigma^2.**

Reference: With df=6, lower-tail chi-square quantiles q_L=1.6353829 and q_U=12.5915872, use ((df)s^2/q_U, (df)s^2/q_L). Denominators reverse.

Answer: (4.289, 33.02)

1. df=6; printed q_L=1.6353829 and q_U=12.5915872.
2. Lower=6(9)/12.5915872=4.289; upper=6(9)/1.6353829=33.02.

2. **A normal population has known sigma=5. A sample of n=8 has mean 43. Construct a 95% z confidence interval for mu.**

Reference: Known population sigma: x-bar +/- z_(1-alpha/2) sigma/sqrt(n). Lower-tail reference z_(1-alpha/2)=1.9599639.

Answer: (39.535, 46.465)

1. Printed z_(1-alpha/2)=1.9599639. Margin=1.9599639(5)/sqrt(8)=3.465.
2. Interval=43 +/- 3.465=(39.535, 46.465).

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

3. **For known sigma=6, n=9, and x-bar=46, invert the two-sided level 0.05 z test of H0: mu=mu0. Give all mu0 values not rejected.**

Reference: The two-sided z test keeps H0 when |(x-bar-mu0)/(sigma/sqrt(n))|<=z_0.975, with z_0.975=1.9599639. Solving gives the matching 95% confidence interval.

Answer: mu0 in [42.08, 49.92]

1. Printed z_0.975=1.9599639; solve |46-mu0|<=1.9599639(6)/sqrt(9).
2. The nonrejected values are [42.08, 49.92], including both endpoints under the stated <= rule.

4. **Ten repeated-sample intervals for the same parameter are (66,70), (66,72), (75,79), (65,71), (67,71), (67,73), (66,70), (73,79), (68,72), (65,71). The true value is 69. Find observed coverage and interpret it.**

Reference: An interval covers when lower <= true value <= upper. Long-run confidence concerns the procedure across repeated samples, not a probability assigned to this fixed parameter.

Answer: 8/10 = 80%

1. Count intervals containing 69: 8 of 10.
2. This small run has 80% observed coverage; it can differ substantially from the nominal level.

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

5. **A random sample from a normal population has n=11, x-bar=52, and sample SD s=7; population variance is unknown. Construct a 95% t confidence interval for mu.**

Reference: Unknown population sigma with normal data: x-bar +/- t_(1-alpha/2,n-1)s/sqrt(n). Lower-tail reference t_(1-alpha/2,10)=2.2281389.

Answer: (47.297, 56.703)

1. df=10; printed t_(1-alpha/2,df)=2.2281389.
2. Margin=2.2281389(7)/sqrt(11)=4.703; interval=(47.297, 56.703).

6. **A student proposes x-bar +/- 1.5s/sqrt(n) as a data-based interval, where mu is the unknown population mean. Is it an interval statistic computable before mu is known?**

Reference: An interval statistic may use observations and known constants, but its endpoints cannot contain the unknown target parameter.

Answer: Yes; its endpoints use sample statistics only

1. x-bar, s, and n are computable from the sample; no unknown population parameter appears.

Name _____ Score _____ Teacher _____ Date _____

Normal-Data Confidence Intervals

Worked Answer Key

Use lower-tail printed quantiles. State whether population variance is known, unknown, or itself the target.

7. **Evaluate this statement: “There is a 86% probability that this fixed mu lies in the computed frequentist interval.”**

Reference: A frequentist confidence level is long-run procedure coverage. A Bayesian credible probability is conditional on a stated prior and model.

Answer: Incorrect as a frequentist interpretation

1. After data are observed, the frequentist endpoints and fixed parameter are not random under the repeated-sampling interpretation.

8. **A normal population has known sigma=11. A sample of n=14 has mean 61. Construct a 95% z confidence interval for mu.**

Reference: Known population sigma: x-bar +/- z_(1-alpha/2) sigma/sqrt(n). Lower-tail reference z_(1-alpha/2)=1.9599639.

Answer: (55.238, 66.762)

1. Printed z_(1-alpha/2)=1.9599639. Margin=1.9599639(11)/sqrt(14)=5.762.
2. Interval=61 +/- 5.762=(55.238, 66.762).