

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

- 1. Prior $\theta \sim N(4, 2)$. A sample of $n=2$ observations has mean 1; each observation has known variance 8. Find the posterior and compare its mean and variance with the prior and data.**

Given: The sample mean has variance observation variance divided by n , so its precision is n/σ^2 .

- 2. Prior $\theta \sim N(5, 4)$. A sample of $n=3$ observations has mean 3; each observation has known variance 1. Find the posterior mean and variance.**

Given: The sample mean has variance observation variance divided by n , so its precision is n/σ^2 .

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

3. Prior $\theta \sim N(6, 8)$. One observation $x=6$ has known variance 2. Find the posterior mean and variance.

Given: The second normal parameter shown is variance. Observation variance is known.

4. Prior $\theta \sim N(1, 1)$; observations 18, 20 each have known variance 4. Update sequentially and verify the batch result.

Given: Observations are conditionally independent given θ .

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

- 5. Prior $\theta \sim N(2, 2)$. A sample of $n=6$ observations has mean 3; each observation has known variance 8. Find the posterior mean and variance.**

Given: The sample mean has variance observation variance divided by n , so its precision is n/σ^2 .

- 6. Prior $\theta \sim N(3, 4)$. A sample of $n=2$ observations has mean 5; each observation has known variance 1. Find the posterior and compare its mean and variance with the prior and data.**

Given: The sample mean has variance observation variance divided by n , so its precision is n/σ^2 .

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

7. Prior $\theta \sim N(4, 8)$. One observation $x=3$ has known variance 2. Find the posterior mean and variance.

Given: The second normal parameter shown is variance. Observation variance is known.

8. Prior $\theta \sim N(5, 1)$; observations 22, 24 each have known variance 4. Update sequentially and verify the batch result.

Given: Observations are conditionally independent given θ .

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

Worked Answer Key

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

1. Prior $\theta \sim N(4, 2)$. A sample of $n=2$ observations has mean 1; each observation has known variance 8. Find the posterior and compare its mean and variance with the prior and data.

Answer: Posterior $N(3, 1.3333)$

1. Prior precision=0.5; data precision=2/8=0.25.
2. Posterior mean=3 lies between prior mean 4 and sample mean 1.
3. Posterior variance 1.3333 is smaller than prior variance 2.

2. Prior $\theta \sim N(5, 4)$. A sample of $n=3$ observations has mean 3; each observation has known variance 1. Find the posterior mean and variance.

Answer: Posterior $N(3.1538, 0.3077)$

1. Prior precision=0.25; data precision=3/1=3.
2. Posterior mean=3.1538 lies between prior mean 5 and sample mean 3.
3. Posterior variance 0.3077 is smaller than prior variance 4.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

Worked Answer Key

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

3. Prior $\theta \sim N(6, 8)$. One observation $x=6$ has known variance 2. Find the posterior mean and variance.

Answer: $N(6, 1.6)$

1. Prior precision= $1/8$; data precision= $1/2$.
2. Posterior variance= $1/(\text{precision sum})=1.6$.
3. Posterior mean is the precision-weighted average 6.

4. Prior $\theta \sim N(1, 1)$; observations 18, 20 each have known variance 4. Update sequentially and verify the batch result.

Answer: Sequential and batch posterior: $N(7, 0.6667)$

1. Sequential updates give $N(7, 0.6667)$.
2. Batch sample mean=19 with $n=2$.
3. The batch update gives $N(7, 0.6667)$, matching sequentially.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

Worked Answer Key

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

5. Prior $\theta \sim N(2, 2)$. A sample of $n=6$ observations has mean 3; each observation has known variance 8. Find the posterior mean and variance.

Answer: Posterior $N(2.6, 0.8)$

1. Prior precision=0.5; data precision=6/8=0.75.
2. Posterior mean=2.6 lies between prior mean 2 and sample mean 3.
3. Posterior variance 0.8 is smaller than prior variance 2.

6. Prior $\theta \sim N(3, 4)$. A sample of $n=2$ observations has mean 5; each observation has known variance 1. Find the posterior and compare its mean and variance with the prior and data.

Answer: Posterior $N(4.7778, 0.4444)$

1. Prior precision=0.25; data precision=2/1=2.
2. Posterior mean=4.7778 lies between prior mean 3 and sample mean 5.
3. Posterior variance 0.4444 is smaller than prior variance 4.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Normal Conjugate Priors

Worked Answer Key

The unknown mean has a normal prior and observations have known variance. Add precisions: prior precision $1/\tau^2$ and data precision n/σ^2 . Use the supplied sample mean and keep variance distinct from standard deviation.

7. Prior $\theta \sim N(4, 8)$. One observation $x=3$ has known variance 2. Find the posterior mean and variance.

Answer: $N(3.2, 1.6)$

1. Prior precision= $1/8$; data precision= $1/2$.
2. Posterior variance= $1/(\text{precision sum})=1.6$.
3. Posterior mean is the precision-weighted average 3.2.

8. Prior $\theta \sim N(5, 1)$; observations 22, 24 each have known variance 4. Update sequentially and verify the batch result.

Answer: Sequential and batch posterior: $N(11, 0.6667)$

1. Sequential updates give $N(11, 0.6667)$.
2. Batch sample mean=23 with $n=2$.
3. The batch update gives $N(11, 0.6667)$, matching sequentially.