

Name _____ Score _____ Teacher _____ Date _____

Multiple Regression Introduction

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

- 1.** In $\hat{y} = 5 + 1x_1 - 1x_2$, interpret coefficient 1 when x_1 is study hours, x_2 is prior practice score, and y is predicted quiz points.

Reference: A multiple-regression slope compares predictions while holding the other included predictor fixed. It is an association unless causal conditions are separately justified.

- 2.** Using $\hat{y} = 6 + 2x_1 - 1x_2$, calculate residuals and SSE for rows (x_1, x_2, y) : (1, 0, 9), (2, 2, 7), (3, 4, 8), (4, 1, 14), (5, 3, 12).

Reference: For each printed row use residual $y - \hat{y}$. Then $SSE = \sum \text{residual}^2$. Use the supplied coefficients; do not refit them.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Multiple Regression Introduction

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

- 3.** For rows (x_1, x_2, y) : (1, 1, 5), (2, 3, 7), (3, 0, 14), (4, 2, 13), (5, 4, 15), (6, 1, 22), write the least-squares objective in b_0, b_1, b_2 for $\hat{y} = b_0 + b_1x_1 + b_2x_2$. Do not solve it.

Reference: Substitute each row into $\sum [y - (b_0 + b_1x_1 + b_2x_2)]^2$. Keep every one of the 4-6 printed rows.

- 4.** A model predicts weekly plant growth y (cm) from sunlight x_1 (hours/day) and water x_2 (liters/week): $\hat{y} = 5 + 4x_1 - 1x_2$. Identify the response and both predictors.

Reference: The response is the outcome on the left side. Predictors appear on the right side and are used to form predictions.

Name _____ Score _____ Teacher _____ Date _____

Multiple Regression Introduction

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

- 5.** For $\hat{y} = 4 + 1x_1 - 2x_2$ and observed row $(x_1, x_2, y) = (1, 0, 4)$, calculate the prediction and residual $\hat{y}$.

Reference: Substitute both predictors into the supplied equation. Residual = observed - predicted, so its sign matters.

- 6.** For $\hat{y} = 5 + 2x_1 - 2x_2$ and observed row $(x_1, x_2, y) = (1, 1, 5)$, calculate the prediction and residual $\hat{y}$.

Reference: Substitute both predictors into the supplied equation. Residual = observed - predicted, so its sign matters.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Multiple Regression Introduction

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

- 7.** In $\hat{y} = 6 + 3x_1 - 2x_2$, interpret coefficient 3 when x_1 is training sessions, x_2 is baseline time, and y is predicted race time seconds.

Reference: A multiple-regression slope compares predictions while holding the other included predictor fixed. It is an association unless causal conditions are separately justified.

- 8.** For rows (x_1, x_2, y) : (1, 3, 1), (2, 0, 12), (3, 2, 13), (4, 4, 11), (5, 1, 22), write the least-squares objective in b_0, b_1, b_2 for $\hat{y} = b_0 + b_1x_1 + b_2x_2$. Do not solve it.

Reference: Substitute each row into $\sum [y - (b_0 + b_1x_1 + b_2x_2)]^2$. Keep every one of the 4-6 printed rows.

Multiple Regression Introduction

Worked Answer Key

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

1. In $\hat{y} = 5 + 1x_1 - 1x_2$, interpret coefficient 1 when x_1 is study hours, x_2 is prior practice score, and y is predicted quiz points.

Reference: A multiple-regression slope compares predictions while holding the other included predictor fixed. It is an association unless causal conditions are separately justified.

Answer: Holding prior practice score fixed, one additional unit of study hours is associated with 1 more predicted units of quiz points

1. Compare two rows with the same x_2 whose x_1 values differ by 1: predictions differ by 1.

2. This coefficient statement alone is not a causal claim and is not a test of significance.

2. Using $\hat{y} = 6 + 2x_1 - 1x_2$, calculate residuals and SSE for rows (x_1, x_2, y) : (1, 0, 9), (2, 2, 7), (3, 4, 8), (4, 1, 14), (5, 3, 12).

Reference: For each printed row use residual $y - \hat{y}$. Then $SSE = \sum \text{residual}^2$. Use the supplied coefficients; do not refit them.

Answer: residuals 1, -1, 0, 1, -1; SSE=4

1. (1,0,9): $\hat{y} = 8$, residual=1.

2. (2,2,7): $\hat{y} = 8$, residual=-1.

3. (3,4,8): $\hat{y} = 8$, residual=0.

4. (4,1,14): $\hat{y} = 13$, residual=1.

5. (5,3,12): $\hat{y} = 13$, residual=-1.

Multiple Regression Introduction

Worked Answer Key

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

3.

For rows (x1,x2,y): (1, 1, 5), (2, 3, 7), (3, 0, 14), (4, 2, 13), (5, 4, 15), (6, 1, 22), write the least-squares objective in b0,b1,b2 for yhat=b0+b1x1+b2x2. Do not solve it.

Reference: Substitute each row into sum[y-(b0+b1 x1+b2 x2)]^2. Keep every one of the 4-6 printed rows.

Answer: [5-(b0+1b1+1b2)]^2 + [7-(b0+2b1+3b2)]^2 + [14-(b0+3b1+0b2)]^2 + [13-(b0+4b1+2b2)]^2 + [15-(b0+5b1+4b2)]^2 + [22-(b0+6b1+1b2)]^2

1. Each bracket is observed y minus the model prediction for one row.

2. The objective has 6 squared terms and is minimized over b0,b1,b2.

4.

A model predicts weekly plant growth y (cm) from sunlight x1 (hours/day) and water x2 (liters/week): yhat=5+4x1-1x2. Identify the response and both predictors.

Reference: The response is the outcome on the left side. Predictors appear on the right side and are used to form predictions.

Answer: Response: weekly growth y. Predictors: sunlight x1 and water x2.

1. Units: y is centimeters, x1 is hours/day, and x2 is liters/week.

2. The equation supplies coefficients; it does not prove either predictor causes growth.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Multiple Regression Introduction

Worked Answer Key

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

5. For $\hat{y} = 4 + 1x_1 - 2x_2$ and observed row $(x_1, x_2, y) = (1, 0, 4)$, calculate the prediction and residual $y - \hat{y}$.

Reference: Substitute both predictors into the supplied equation. Residual = observed - predicted, so its sign matters.

Answer: $\hat{y} = 5$; residual = -1

1. $\hat{y} = 4 + 1(1) - 2(0) = 5$.
2. Residual = $4 - 5 = -1$.

6. For $\hat{y} = 5 + 2x_1 - 2x_2$ and observed row $(x_1, x_2, y) = (1, 1, 5)$, calculate the prediction and residual $y - \hat{y}$.

Reference: Substitute both predictors into the supplied equation. Residual = observed - predicted, so its sign matters.

Answer: $\hat{y} = 5$; residual = 0

1. $\hat{y} = 5 + 2(1) - 2(1) = 5$.
2. Residual = $5 - 5 = 0$.

Multiple Regression Introduction

Worked Answer Key

Use the supplied coefficients. Interpret one slope while holding the other predictor fixed; no coefficient inference is required.

7. In $\hat{y} = 6 + 3x_1 - 2x_2$, interpret coefficient 3 when x_1 is training sessions, x_2 is baseline time, and y is predicted race time seconds.

Reference: A multiple-regression slope compares predictions while holding the other included predictor fixed. It is an association unless causal conditions are separately justified.

Answer: Holding baseline time fixed, one additional unit of training sessions is associated with 3 more predicted units of race time seconds

1. Compare two rows with the same x_2 whose x_1 values differ by 1: predictions differ by 3.

2. This coefficient statement alone is not a causal claim and is not a test of significance.

8. For rows (x_1, x_2, y) : (1, 3, 1), (2, 0, 12), (3, 2, 13), (4, 4, 11), (5, 1, 22), write the least-squares objective in b_0, b_1, b_2 for $\hat{y} = b_0 + b_1x_1 + b_2x_2$. Do not solve it.

Reference: Substitute each row into $\sum (y - (b_0 + b_1x_1 + b_2x_2))^2$. Keep every one of the 4-6 printed rows.

Answer: $[1 - (b_0 + 1b_1 + 3b_2)]^2 + [12 - (b_0 + 2b_1 + 0b_2)]^2 + [13 - (b_0 + 3b_1 + 2b_2)]^2 + [11 - (b_0 + 4b_1 + 4b_2)]^2 + [22 - (b_0 + 5b_1 + 1b_2)]^2$

1. Each bracket is observed y minus the model prediction for one row.

2. The objective has 5 squared terms and is minimized over b_0, b_1, b_2 .