

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

1. An ordered Bernoulli sequence produced 2 successes in 8 fixed trials. Compare $p = 0.3$ by likelihood and choose the best listed value.

Given: The observed data are fixed while p varies. A binomial coefficient is common to all listed candidates and does not change their ranking.

p	relative likelihood
0.2	0.010486
0.35	0.009239

2. The total of 4 independent exponential waiting times is 12 hours. Find the MLE of the rate λ in events per hour.

Given: Waiting times are independent exponential(λ), $\lambda > 0$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

3. A binomial count has 4 successes and 4 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Given: Trials are independent with a common unknown success probability $0 \leq p \leq 1$.

4. Data 40, 41, 42, 43 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 1$. Find the MLE and explain the support endpoint.

Given: The likelihood is $\theta^{-(n)}$ when θ is at least every observation, and 0 otherwise.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

5. An ordered Bernoulli sequence has 5 successes and 5 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Given: Trials are independent with a common unknown success probability $0 \leq p \leq 1$.

6. Find the MLE of p after 5 successes in 11 independent Bernoulli trials. Check boundary cases.

Given: The parameter support is $0 \leq p \leq 1$.

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

7. 5 fixed exponential waiting times total 12 hours. Compare candidate rates $\lambda = 1, 1.5$ events per hour and choose the best listed value.

Given: For fixed nonnegative waiting times, $L(\lambda)$ is proportional to $\lambda^n \exp(-\lambda \text{ times the sample sum})$, with $\lambda > 0$.

λ	relative likelihood
1	0.000006
1.5	0.000000

8. Data 74, 75, 76, 77 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 0$. Find the MLE and explain the support endpoint.

Given: The likelihood is $\theta^{-(n)}$ when θ is at least every observation, and 0 otherwise.