

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

1. An ordered Bernoulli sequence produced 2 successes in 8 fixed trials. Compare $p = 0.3$ by likelihood and choose the best listed value.

Given: The observed data are fixed while p varies. A binomial coefficient is common to all listed candidates and does not change their ranking.

p	relative likelihood
0.2	0.010486
0.35	0.009239

2. The total of 4 independent exponential waiting times is 12 hours. Find the MLE of the rate λ in events per hour.

Given: Waiting times are independent $\text{exponential}(\lambda)$, $\lambda > 0$.

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

3. A binomial count has 4 successes and 4 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Given: Trials are independent with a common unknown success probability $0 \leq p \leq 1$.

4. Data 40, 41, 42, 43 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 1$. Find the MLE and explain the support endpoint.

Given: The likelihood is $\theta^{-(n)}$ when θ is at least every observation, and 0 otherwise.

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

5. An ordered Bernoulli sequence has 5 successes and 5 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Given: Trials are independent with a common unknown success probability $0 \leq p \leq 1$.

6. Find the MLE of p after 5 successes in 11 independent Bernoulli trials. Check boundary cases.

Given: The parameter support is $0 \leq p \leq 1$.

Likelihood and Maximum Likelihood

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

7. 5 fixed exponential waiting times total 12 hours. Compare candidate rates $\lambda = 1, 1.5$ events per hour and choose the best listed value.

Given: For fixed nonnegative waiting times, $L(\lambda)$ is proportional to $\lambda^n \exp(-\lambda \text{ times the sample sum})$, with $\lambda > 0$.

λ	relative likelihood
1	0.000006
1.5	0.000000

8. Data 74, 75, 76, 77 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 0$. Find the MLE and explain the support endpoint.

Given: The likelihood is $\theta^{-(n)}$ when θ is at least every observation, and 0 otherwise.

Likelihood and Maximum Likelihood

Worked Answer Key

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

1. An ordered Bernoulli sequence produced 2 successes in 8 fixed trials. Compare $p = 0.2, 0.3$ by likelihood and choose the best listed value.

p	relative likelihood
0.2	0.010486
0.35	0.009239

- Answer: $p = 0.2$
- Use $L(p)$ proportional to $p^2(1-p)^6$.
 - $L(0.2)$ is proportional to 0.010486.
 - $L(0.35)$ is proportional to 0.009239.
 - The largest listed likelihood occurs at $p=0.2$.

2. The total of 4 independent exponential waiting times is 12 hours. Find the MLE of the rate λ in events per hour.

- Answer: $\lambda\text{-hat} = 1/3$ events per hour
- $\log L(\lambda) = 4\log \lambda - 12\lambda$.
 - Set $4/\lambda - 12 = 0$.
 - $\lambda\text{-hat} = 4/12 = 0.3333$ events per hour.

Likelihood and Maximum Likelihood

Worked Answer Key

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

3. A binomial count has 4 successes and 4 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Answer: $L(p) = C(8,4) p^4(1-p)^4$

- $\log L(p) = \log C(8,4) + 4 \log p + 4 \log(1-p)$.
- The data counts are fixed while p varies.

4. Data 40, 41, 42, 43 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 1$. Find the MLE and explain the support endpoint.

Answer: $\theta\text{-hat} = 43$

- The support condition requires $\theta \geq \max(\text{data}) = 43$.
- On the allowed support, θ^{-4} decreases as θ increases.
- Choose the smallest allowed θ , 43.

Name _____ Score _____ Teacher _____ Date _____

Likelihood and Maximum Likelihood

Worked Answer Key

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

5. An ordered Bernoulli sequence has 5 successes and 5 failures. Write $L(p)$ and $\log L(p)$, including terms that depend on p .

Answer: $L(p) = 1 p^5(1-p)^5$

1. $\log L(p) = 5 \log p + 5 \log(1-p)$.
2. The data counts are fixed while p varies.

6. Find the MLE of p after 5 successes in 11 independent Bernoulli trials. Check boundary cases.

Answer: $\hat{p} = 5/11 = 0.4545$

1. Differentiate $\log L(p) = 5 \log p + 6 \log(1-p)$.
2. The interior solution is successes/trials; at all-failure or all-success data the maximum is the boundary 0 or 1.
3. Here $\hat{p} = 5/11$.

Likelihood and Maximum Likelihood

Worked Answer Key

Keep the observed data fixed while treating likelihood as a function of the parameter. State the parameter support, use natural log-likelihood when requested, check endpoints or support restrictions, and distinguish the MLE variance divisor n from the sample-variance divisor $n-1$.

7. 5 fixed exponential waiting times total 12 hours. Compare candidate rates $\lambda = 1, 1.5$ events per hour and choose the best listed value.

λ	relative likelihood
1	0.000006
1.5	0.000000

Answer: $\lambda = 1$ events per hour

- 1. Use $L(\lambda)$ proportional to $\lambda^5 \exp(-12\lambda)$.
- 2. $L(1)$ is proportional to 0.000006.
- 3. $L(1.5)$ is proportional to 0.000000.
- 4. The greatest listed likelihood occurs at $\lambda=1$.

8. Data 74, 75, 76, 77 are independent from $\text{Uniform}(0, \theta)$, with $\theta \geq 0$. Find the MLE and explain the support endpoint.

Answer: $\hat{\theta} = 77$

- 1. The support condition requires $\theta \geq \max(\text{data}) = 77$.
- 2. On the allowed support, θ^{-4} decreases as θ increases.
- 3. Choose the smallest allowed θ , 77.