Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Least-Squares Curve Models

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

1. **Is the model yhat=a+b sin(x)+c x^2 linear in its unknown coefficients? Explain what that means for least squares.**

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

2. **For data (-2, 15), (-1, 5), (0, 3), (1, 9), (2, 23), form and solve the normal equations for the least-squares quadratic yhat=a+bx+cx^2.**

Reference: Compute sums n, sum x, sum x^2, sum x^3, sum x^4, sum y, sum xy, and sum x^2y. The equations are [n,sum x,sum x^2; sum x,sum x^2,sum x^3; sum x^2,sum x^3,sum x^4][a,b,c]^T=[sum y,sum xy,sum x^2y]^T.

Show substitutions and residual signs; use only printed values.

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3. **For an exponential model y=A exp(kx), the supplied transformed data (x, ln y) are (0, 0.45), (1, 0.75), (2, 1.05), (3, 1.35), (4, 1.65). Fit the least-squares line ln y=alpha+kx, then give A and k.**

Reference: Use only the printed ln y values. Fit a line to (x,ln y); then A=exp(alpha). Report A to three decimals.

4. **For data (-2, 8), (-1, 6), (0, 4), (1, 6), (2, 8), evaluate SSE for L(x)=4+0.5x, Q(x)=4+x^2, and C(x)=4-0.25x+x^2+0.1x^3. Which supplied model has the smallest SSE?**

Reference: For each row compute residual y-yhat, square it, then sum. SSE=sum(y-yhat)^2. Use the printed coefficients exactly.

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Least-Squares Curve Models

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

5. **For an exponential model y=A exp(kx), the supplied transformed data (x, ln y) are (0, 0.55), (1, 0.75), (2, 0.95), (3, 1.15), (4, 1.35). Fit the least-squares line ln y=alpha+kx, then give A and k.**

Reference: Use only the printed ln y values. Fit a line to (x,ln y); then A=exp(alpha). Report A to three decimals.

6. **Is the model yhat=a+b x+c cos(x) linear in its unknown coefficients? Explain what that means for least squares.**

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

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Least-Squares Curve Models

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7. **For data (-2, 17), (-1, 4), (0, 1), (1, 8), (2, 25), form and solve the normal equations for the least-squares quadratic yhat=a+bx+cx^2.**

Reference: Compute sums n, sum x, sum x^2, sum x^3, sum x^4, sum y, sum xy, and sum x^2y. The equations are [n,sum x,sum x^2; sum x,sum x^2,sum x^3; sum x^2,sum x^3,sum x^4][a,b,c]^T=[sum y,sum xy,sum x^2y]^T.

8. **For data (-2, 13), (-1, 9), (0, 9), (1, 9), (2, 13), evaluate SSE for L(x)=8+0.5x, Q(x)=8+x^2, and C(x)=8-0.25x+x^2+0.1x^3. Which supplied model has the smallest SSE?**

Reference: For each row compute residual y-yhat, square it, then sum. SSE=sum(y-yhat)^2. Use the printed coefficients exactly.

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Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

1. **Is the model yhat=a+b sin(x)+c x^2 linear in its unknown coefficients? Explain what that means for least squares.**

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

Answer: Yes; it is linear in the coefficients

1. The basis functions are known once x is supplied, so the prediction is a linear combination of coefficients.

2. **For data (-2, 15), (-1, 5), (0, 3), (1, 9), (2, 23), form and solve the normal equations for the least-squares quadratic yhat=a+bx+cx^2.**

Reference: Compute sums n, sum x, sum x^2, sum x^3, sum x^4, sum y, sum xy, and sum x^2y. The equations are [n,sum x,sum x^2; sum x,sum x^2,sum x^3; sum x^2,sum x^3,sum x^4][a,b,c]^T=[sum y,sum xy,sum x^2y]^T.

Answer: Equations: [5, 0, 10] [a,b,c]^T=55; [0, 10, 0] [a,b,c]^T=20; [10, 0, 34] [a,b,c]^T=166. Solution a=3, b=2, c=4.

1. Sums: n=5, sum x=0, sum x^2=10, sum x^3=0, sum x^4=34, sum y=55, sum xy=20, sum x^2y=166.
2. The formed equations are 5a+0b+10c=55; 0a+10b+0c=20; 10a+0b+34c=166. Row reduction gives (3, 2, 4).

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Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

3. **For an exponential model y=A exp(kx), the supplied transformed data (x, ln y) are (0, 0.45), (1, 0.75), (2, 1.05), (3, 1.35), (4, 1.65). Fit the least-squares line ln y=alpha+kx, then give A and k.**

Reference: Use only the printed ln y values. Fit a line to (x,ln y); then A=exp(alpha). Report A to three decimals.

Answer: alpha=0.45, k=0.3, A=1.568

1. The transformed points lie on ln y=0.45+0.3x.
2. A=exp(0.45)=1.568; hence y=1.568 exp(0.3x).

4. **For data (-2, 8), (-1, 6), (0, 4), (1, 6), (2, 8), evaluate SSE for L(x)=4+0.5x, Q(x)=4+x^2, and C(x)=4-0.25x+x^2+0.1x^3. Which supplied model has the smallest SSE?**

Reference: For each row compute residual y-yhat, square it, then sum. SSE=sum(y-yhat)^2. Use the printed coefficients exactly.

Answer: SSE(L)=42.5, SSE(Q)=2, SSE(C)=2.225; Q is smallest

1. L residuals: 5, 2.5, 0, 1.5, 3; SSE=42.5.
2. Q residuals: 0, 1, 0, 1, 0; SSE=2.
3. C residuals: 0.3, 0.85, 0, 1.15, -0.3; SSE=2.225.

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Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

5. **For an exponential model y=A exp(kx), the supplied transformed data (x, ln y) are (0, 0.55), (1, 0.75), (2, 0.95), (3, 1.15), (4, 1.35). Fit the least-squares line ln y=alpha+kx, then give A and k.**

Reference: Use only the printed ln y values. Fit a line to (x,ln y); then A=exp(alpha). Report A to three decimals.

Answer: alpha=0.55, k=0.2, A=1.733

1. The transformed points lie on ln y=0.55+0.2x.
2. A=exp(0.55)=1.733; hence y=1.733 exp(0.2x).

6. **Is the model yhat=a+b x+c cos(x) linear in its unknown coefficients? Explain what that means for least squares.**

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

Answer: Yes; it is linear in the coefficients

1. The basis functions are known once x is supplied, so the prediction is a linear combination of coefficients.

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Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

7. **For data (-2, 17), (-1, 4), (0, 1), (1, 8), (2, 25), form and solve the normal equations for the least-squares quadratic yhat=a+bx+cx^2.**

Reference: Compute sums n, sum x, sum x^2, sum x^3, sum x^4, sum y, sum xy, and sum x^2y. The equations are [n,sum x,sum x^2; sum x,sum x^2,sum x^3; sum x^2,sum x^3,sum x^4][a,b,c]^T=[sum y,sum xy,sum x^2y]^T.

Answer: Equations: [5, 0, 10] [a,b,c]^T=55; [0, 10, 0] [a,b,c]^T=20; [10, 0, 34] [a,b,c]^T=180. Solution a=1, b=2, c=5.

1. Sums: n=5, sum x=0, sum x^2=10, sum x^3=0, sum x^4=34, sum y=55, sum xy=20, sum x^2y=180.
2. The formed equations are 5a+0b+10c=55; 0a+10b+0c=20; 10a+0b+34c=180. Row reduction gives (1, 2, 5).

8. **For data (-2, 13), (-1, 9), (0, 9), (1, 9), (2, 13), evaluate SSE for L(x)=8+0.5x, Q(x)=8+x^2, and C(x)=8-0.25x+x^2+0.1x^3. Which supplied model has the smallest SSE?**

Reference: For each row compute residual y-yhat, square it, then sum. SSE=sum(y-yhat)^2. Use the printed coefficients exactly.

Answer: SSE(L)=55.5, SSE(Q)=3, SSE(C)=3.225; Q is smallest

1. L residuals: 6, 1.5, 1, 0.5, 4; SSE=55.5.
2. Q residuals: 1, 0, 1, 0, 1; SSE=3.
3. C residuals: 1.3, -0.15, 1, 0.15, 0.7; SSE=3.225.