

Least-Squares Curve Models

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

1. Is the model $\hat{y} = a + b \sin(x) + c x^2$ linear in its unknown coefficients? Explain what that means for least squares.

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

2. For data $(-2, 15), (-1, 5), (0, 3), (1, 9), (2, 23)$, form and solve the normal equations for the least-squares quadratic $\hat{y} = a + bx + cx^2$.

Reference: Compute sums $n, \sum x, \sum x^2, \sum x^3, \sum x^4, \sum y, \sum xy,$ and $\sum x^2y$. The equations are $[n, \sum x, \sum x^2; \sum x, \sum x^2, \sum x^3; \sum x^2, \sum x^3, \sum x^4][a, b, c]^T = [\sum y, \sum xy, \sum x^2y]^T$.

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3. For an exponential model $y=A \exp(kx)$, the supplied transformed data $(x, \ln y)$ are $(0, 0.45), (1, 0.75), (2, 1.05), (3, 1.35), (4, 1.65)$. Fit the least-squares line $\ln y=\alpha+kx$, then give A and k.

Reference: Use only the printed $\ln y$ values. Fit a line to $(x,\ln y)$; then $A=\exp(\alpha)$. Report A to three decimals.

4. For data $(-2, 8), (-1, 6), (0, 4), (1, 6), (2, 8)$, evaluate SSE for $L(x)=4+0.5x$, $Q(x)=4+x^2$, and $C(x)=4-0.25x+x^2+0.1x^3$. Which supplied model has the smallest SSE?

Reference: For each row compute residual $y-\hat{y}$, square it, then sum. $SSE=\sum(y-\hat{y})^2$. Use the printed coefficients exactly.

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5. For an exponential model $y=A \exp(kx)$, the supplied transformed data $(x, \ln y)$ are $(0, 0.55), (1, 0.75), (2, 0.95), (3, 1.15), (4, 1.35)$. Fit the least-squares line $\ln y=\alpha+kx$, then give A and k.

Reference: Use only the printed $\ln y$ values. Fit a line to $(x,\ln y)$; then $A=\exp(\alpha)$. Report A to three decimals.

6. Is the model $\hat{y}=a+b x+c \cos (x)$ linear in its unknown coefficients? Explain what that means for least squares.

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

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7.

For data (-2, 17), (-1, 4), (0, 1), (1, 8), (2, 25), form and solve the normal equations for the least-squares quadratic $\hat{y}=a+bx+cx^2$.

Reference: Compute sums n , $\sum x$, $\sum x^2$, $\sum x^3$, $\sum x^4$, $\sum y$, $\sum xy$, and $\sum x^2y$. The equations are $[n,\sum x,\sum x^2; \sum x,\sum x^2,\sum x^3; \sum x^2,\sum x^3,\sum x^4][a,b,c]^T=[\sum y,\sum xy,\sum x^2y]^T$.

8.

For data (-2, 13), (-1, 9), (0, 9), (1, 9), (2, 13), evaluate SSE for $L(x)=8+0.5x$, $Q(x)=8+x^2$, and $C(x)=8-0.25x+x^2+0.1x^3$. Which supplied model has the smallest SSE?

Reference: For each row compute residual $y-\hat{y}$, square it, then sum. $SSE=\sum (y-\hat{y})^2$. Use the printed coefficients exactly.

Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

1.

Is the model $\hat{y}=a+b \sin(x)+c x^2$ linear in its unknown coefficients? Explain what that means for least squares.

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

Answer: Yes; it is linear in the coefficients

1. The basis functions are known once x is supplied, so the prediction is a linear combination of coefficients.

2.

For data $(-2, 15), (-1, 5), (0, 3), (1, 9), (2, 23)$, form and solve the normal equations for the least-squares quadratic $\hat{y}=a+bx+cx^2$.

Reference: Compute sums $n, \sum x, \sum x^2, \sum x^3, \sum x^4, \sum y, \sum xy$, and $\sum x^2y$. The equations are $[n, \sum x, \sum x^2; \sum x, \sum x^2, \sum x^3; \sum x^2, \sum x^3, \sum x^4][a, b, c]^T = [\sum y, \sum xy, \sum x^2y]^T$.

Answer: Equations: $[5, 0, 10] [a, b, c]^T=55$; $[0, 10, 0] [a, b, c]^T=20$; $[10, 0, 34] [a, b, c]^T=166$. Solution $a=3, b=2, c=4$.

1. Sums: $n=5, \sum x=0, \sum x^2=10, \sum x^3=0, \sum x^4=34, \sum y=55, \sum xy=20, \sum x^2y=166$.

2. The formed equations are $5a+0b+10c=55$; $0a+10b+0c=20$; $10a+0b+34c=166$. Row reduction gives $(3, 2, 4)$.

Least-Squares Curve Models

Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

3.

For an exponential model $y=A \exp(kx)$, the supplied transformed data $(x, \ln y)$ are $(0, 0.45), (1, 0.75), (2, 1.05), (3, 1.35), (4, 1.65)$. Fit the least-squares line $\ln y=\alpha+kx$, then give A and k.

Reference: Use only the printed $\ln y$ values. Fit a line to $(x,\ln y)$; then $A=\exp(\alpha)$. Report A to three decimals.

Answer: $\alpha=0.45, k=0.3, A=1.568$

1. The transformed points lie on $\ln y=0.45+0.3x$.

2. $A=\exp(0.45)=1.568$; hence $y=1.568 \exp(0.3x)$.

4.

For data $(-2, 8), (-1, 6), (0, 4), (1, 6), (2, 8)$, evaluate SSE for $L(x)=4+0.5x, Q(x)=4+x^2$, and $C(x)=4-0.25x+x^2+0.1x^3$. Which supplied model has the smallest SSE?

Reference: For each row compute residual $y-\hat{y}$, square it, then sum. $SSE=\sum(y-\hat{y})^2$. Use the printed coefficients exactly.

Answer: $SSE(L)=42.5, SSE(Q)=2, SSE(C)=2.225$; Q is smallest

1. L residuals: 5, 2.5, 0, 1.5, 3; $SSE=42.5$.

2. Q residuals: 0, 1, 0, 1, 0; $SSE=2$.

3. C residuals: 0.3, 0.85, 0, 1.15, -0.3; $SSE=2.225$.

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5.

For an exponential model $y=A \exp(kx)$, the supplied transformed data $(x, \ln y)$ are $(0, 0.55), (1, 0.75), (2, 0.95), (3, 1.15), (4, 1.35)$. Fit the least-squares line $\ln y=\alpha+kx$, then give A and k.

Reference: Use only the printed $\ln y$ values. Fit a line to $(x,\ln y)$; then $A=\exp(\alpha)$. Report A to three decimals.

Answer: $\alpha=0.55$, $k=0.2$, $A=1.733$

1. The transformed points lie on $\ln y=0.55+0.2x$.

2. $A=\exp(0.55)=1.733$; hence $y=1.733 \exp(0.2x)$.

6.

Is the model $\hat{y}=a+b x+c \cos(x)$ linear in its unknown coefficients? Explain what that means for least squares.

Reference: A model is linear in coefficients when each unknown coefficient multiplies a known function of x and coefficients are not multiplied together or placed inside nonlinear functions.

Answer: Yes; it is linear in the coefficients

1. The basis functions are known once x is supplied, so the prediction is a linear combination of coefficients.

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Worked Answer Key

Use every printed data value and supplied model. Compare models by SSE, not by visual preference alone.

7. For data $(-2, 17), (-1, 4), (0, 1), (1, 8), (2, 25)$, form and solve the normal equations for the least-squares quadratic $\hat{y} = a + bx + cx^2$.

Reference: Compute sums $n, \sum x, \sum x^2, \sum x^3, \sum x^4, \sum y, \sum xy$, and $\sum x^2y$. The equations are $[n, \sum x, \sum x^2; \sum x, \sum x^2, \sum x^3; \sum x^2, \sum x^3, \sum x^4][a, b, c]^T = [\sum y, \sum xy, \sum x^2y]^T$.

Answer: Equations: $[5, 0, 10] [a, b, c]^T = 55; [0, 10, 0] [a, b, c]^T = 20; [10, 0, 34] [a, b, c]^T = 180$. **Solution** $a=1, b=2, c=5$.

1. Sums: $n=5, \sum x=0, \sum x^2=10, \sum x^3=0, \sum x^4=34, \sum y=55, \sum xy=20, \sum x^2y=180$.
2. The formed equations are $5a+0b+10c=55; 0a+10b+0c=20; 10a+0b+34c=180$. Row reduction gives $(1, 2, 5)$.

8. For data $(-2, 13), (-1, 9), (0, 9), (1, 9), (2, 13)$, evaluate SSE for $L(x)=8+0.5x, Q(x)=8+x^2$, and $C(x)=8-0.25x+x^2+0.1x^3$. Which supplied model has the smallest SSE?

Reference: For each row compute residual $y-\hat{y}$, square it, then sum. $SSE = \sum (y-\hat{y})^2$. Use the printed coefficients exactly.

Answer: $SSE(L)=55.5, SSE(Q)=3, SSE(C)=3.225$; Q is smallest

1. L residuals: $6, 1.5, 1, 0.5, 4$; $SSE=55.5$.
2. Q residuals: $1, 0, 1, 0, 1$; $SSE=3$.
3. C residuals: $1.3, -0.15, 1, 0.15, 0.7$; $SSE=3.225$.