

Large-Sample Confidence Intervals

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

1. A random independent sample of  $n=64$  observations from a skewed population has  $\bar{x}=20$  and  $s=4$ . Assuming finite variance and that  $n$  is large enough for the CLT approximation, construct a 90% mean interval.

Reference: Approximate large-sample interval:  $\bar{x} \pm z_{(1-\alpha/2)}s/\sqrt{n}$ , with  $z_{(1-\alpha/2)}=1.6448536$ . Independence, finite variance, and adequate CLT accuracy are assumptions.

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2. 31 successes occur in  $n=68$  independent Bernoulli trials. Construct the conservative 95% normal interval using  $p(1-p)\leq 1/4$ .

Reference: Use  $\hat{p} \pm z_{(1-\alpha/2)}/(2\sqrt{n})$ , with  $z_{(1-\alpha/2)}=1.9599639$ ; clip endpoints to  $[0,1]$ . This bound is conservative for the normal-approximation standard error.

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Large-Sample Confidence Intervals

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

3. Two comparable independent proportion studies use  $n=72$  and  $n=288$ . Under the same conservative 95% approximation, compare margin widths and state when nominal coverage may be unreliable.

Reference: Conservative margin is about  $1/\sqrt{n}$ , so multiplying sample size by 4 halves the margin. Normal coverage can be poor with dependence or inadequate success/failure counts.

4. 36 of  $n=76$  independent trials succeed. Use the approximate 95% rule  $\hat{p} \pm 1/\sqrt{n}$ , and label the result as a rule of thumb.

Reference: The  $1/\sqrt{n}$  margin approximates the conservative 95% normal margin because  $1.96/2$  is about 1. It is not exact.

Large-Sample Confidence Intervals

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

5. 39 successes occur in n=80 independent Bernoulli trials. Construct the conservative 95% normal interval using  $p(1-p) \leq 1/4$ .

Reference: Use  $\hat{p} \pm z_{(1-\alpha/2)}/(2\sqrt{n})$ , with  $z_{(1-\alpha/2)}=1.9599639$ ; clip endpoints to  $[0,1]$ . This bound is conservative for the normal-approximation standard error.

6. 42 of n=84 independent trials succeed. Use the approximate 95% rule  $\hat{p} \pm 1/\sqrt{n}$ , and label the result as a rule of thumb.

Reference: The  $1/\sqrt{n}$  margin approximates the conservative 95% normal margin because  $1.96/2$  is about 1. It is not exact.

Large-Sample Confidence Intervals

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

7. A random independent sample of  $n=88$  observations from a skewed population has  $\bar{x}=26$  and  $s=10$ . Assuming finite variance and that  $n$  is large enough for the CLT approximation, construct a 90% mean interval.

Reference: Approximate large-sample interval:  $\bar{x} \pm z_{(1-\alpha/2)}s/\sqrt{n}$ , with  $z_{(1-\alpha/2)}=1.6448536$ . Independence, finite variance, and adequate CLT accuracy are assumptions.

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8. Two comparable independent proportion studies use  $n=92$  and  $n=368$ . Under the same conservative 95% approximation, compare margin widths and state when nominal coverage may be unreliable.

Reference: Conservative margin is about  $1/\sqrt{n}$ , so multiplying sample size by 4 halves the margin. Normal coverage can be poor with dependence or inadequate success/failure counts.

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Large-Sample Confidence Intervals

Worked Answer Key

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

1.

A random independent sample of  $n=64$  observations from a skewed population has  $\bar{x}=20$  and  $s=4$ . Assuming finite variance and that  $n$  is large enough for the CLT approximation, construct a 90% mean interval.

Reference: Approximate large-sample interval:  $\bar{x} \pm z_{(1-\alpha/2)}s/\sqrt{n}$ , with  $z_{(1-\alpha/2)}=1.6448536$ . Independence, finite variance, and adequate CLT accuracy are assumptions.

**Answer: (19.178, 20.822)**

1. Printed  $z=1.6448536$ ; estimated  $SE=4/\sqrt{64}$ .

2. Margin=0.822; approximate interval=(19.178, 20.822).

2.

31 successes occur in  $n=68$  independent Bernoulli trials. Construct the conservative 95% normal interval using  $p(1-p)\leq 1/4$ .

Reference: Use  $\hat{p} \pm z_{(1-\alpha/2)}/(2\sqrt{n})$ , with  $z_{(1-\alpha/2)}=1.9599639$ ; clip endpoints to  $[0,1]$ . This bound is conservative for the normal-approximation standard error.

**Answer: (0.337, 0.575)**

1.  $\hat{p}=31/68=0.456$ ; printed  $z=1.9599639$ .

2. Margin= $1.9599639/(2\sqrt{68})=0.119$ ; interval=(0.337, 0.575).

Large-Sample Confidence Intervals

Worked Answer Key

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

3.

Two comparable independent proportion studies use  $n=72$  and  $n=288$ . Under the same conservative 95% approximation, compare margin widths and state when nominal coverage may be unreliable.

Reference: Conservative margin is about  $1/\sqrt{n}$ , so multiplying sample size by 4 halves the margin. Normal coverage can be poor with dependence or inadequate success/failure counts.

**Answer: margin ratio 2:1; the larger study is half as wide**

1.  $1/\sqrt{72}=0.118$  and  $1/\sqrt{288}=0.059$ .

2. The nominal comparison assumes independent trials and a reasonable normal approximation; it is not a guarantee for rare outcomes or dependent data.

4.

36 of  $n=76$  independent trials succeed. Use the approximate 95% rule  $\hat{p} \pm 1/\sqrt{n}$ , and label the result as a rule of thumb.

Reference: The  $1/\sqrt{n}$  margin approximates the conservative 95% normal margin because  $1.96/2$  is about 1. It is not exact.

**Answer: (0.359, 0.588)**

1.  $\hat{p}=36/76=0.474$ ; margin= $1/\sqrt{76}=0.115$ .

2. Approximate rule-of-thumb interval=(0.359, 0.588).

Large-Sample Confidence Intervals

Worked Answer Key

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

5.

39 successes occur in n=80 independent Bernoulli trials. Construct the conservative 95% normal interval using  $p(1-p) \leq 1/4$ .

Reference: Use  $\hat{p} \pm z_{(1-\alpha/2)}/(2\sqrt{n})$ , with  $z_{(1-\alpha/2)}=1.9599639$ ; clip endpoints to  $[0,1]$ . This bound is conservative for the normal-approximation standard error.

**Answer: (0.378, 0.597)**

1.

$\hat{p}=39/80=0.487$ ; printed  $z=1.9599639$ .

2.

Margin= $1.9599639/(2\sqrt{80})=0.11$ ; interval= $(0.378, 0.597)$ .

6.

42 of n=84 independent trials succeed. Use the approximate 95% rule  $\hat{p} \pm 1/\sqrt{n}$ , and label the result as a rule of thumb.

Reference: The  $1/\sqrt{n}$  margin approximates the conservative 95% normal margin because  $1.96/2$  is about 1. It is not exact.

**Answer: (0.391, 0.609)**

1.

$\hat{p}=42/84=0.5$ ; margin= $1/\sqrt{84}=0.109$ .

2.

Approximate rule-of-thumb interval= $(0.391, 0.609)$ .

Large-Sample Confidence Intervals

Worked Answer Key

State independence and large-sample conditions. Label conservative and rule-of-thumb intervals as approximations.

7.

A random independent sample of  $n=88$  observations from a skewed population has  $\bar{x}=26$  and  $s=10$ . Assuming finite variance and that  $n$  is large enough for the CLT approximation, construct a 90% mean interval.

Reference: Approximate large-sample interval:  $\bar{x} \pm z_{(1-\alpha/2)}s/\sqrt{n}$ , with  $z_{(1-\alpha/2)}=1.6448536$ . Independence, finite variance, and adequate CLT accuracy are assumptions.

**Answer: (24.247, 27.753)**

1. Printed  $z=1.6448536$ ; estimated  $SE=10/\sqrt{88}$ .  
2. Margin= $1.753$ ; approximate interval= $(24.247, 27.753)$ .

8.

Two comparable independent proportion studies use  $n=92$  and  $n=368$ . Under the same conservative 95% approximation, compare margin widths and state when nominal coverage may be unreliable.

Reference: Conservative margin is about  $1/\sqrt{n}$ , so multiplying sample size by 4 halves the margin. Normal coverage can be poor with dependence or inadequate success/failure counts.

**Answer: margin ratio 2:1; the larger study is half as wide**

1.  $1/\sqrt{92}=0.104$  and  $1/\sqrt{368}=0.052$ .  
2. The nominal comparison assumes independent trials and a reasonable normal approximation; it is not a guarantee for rare outcomes or dependent data.