

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

1. For T in $\{0,1,2,3\}$, H_0 has PMF $(0.44,0.31,0.17,0.08)$ and H_A has PMF $(0.08,0.17,0.27,0.48)$. Choose between rejection sets $\{3\}$ and $\{2,3\}$ when alpha may not exceed 0.090. Report the chosen set and its alpha.

Assumptions: The displayed PMFs are complete and outcomes are mutually exclusive.

Reference: For each candidate, alpha is the sum of its probabilities under H_0 .

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

2. A valid test of H0 versus HA reports $p=0.056$ at $\alpha=0.05$. State the decision and write a conclusion that does not treat failure to reject as proof of H0.

Assumptions: The test design and assumptions were checked before the p-value was calculated.

Reference: Reject when $p \leq \alpha$; otherwise fail to reject.

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State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

3. For T in {0,1,2,3}, H0 has PMF (0.46,0.27,0.19,0.08) and HA has PMF (0.10,0.21,0.28,0.41). Reject H0 when T is in {3}. Find alpha, beta, and power.

Assumptions: The displayed hypotheses specify the full sampling distributions.
Reference: alpha=P(reject|H0); beta=P(fail to reject|HA); power=1-beta=P(reject|HA).

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

4. Independent measurements follow $N(\mu, 49)$ with known variance 49. Investigators ask whether the population mean exceeds 54. State H0 and HA, classify each as simple or composite, and name the tail direction.

Assumptions: Measurements are iid normal with unknown fixed mean μ and known $\sigma=7$.

Reference: A one-sided null includes its boundary and all parameter values opposite HA.

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

5. Test H0: $\mu=55$ against HA: $\mu>55$ with known $\sigma=8$, $n=25$, and $\alpha=0.05$. Find the critical sample mean and power at $\mu=59$.

Assumptions: Observations are independent normal measurements and σ is known.

Reference: Normal reference: $z_{(1-\alpha)}=1.644853492$. At $\mu=59$, the printed cutoff standardizes to $z=-0.855146508$, with $P(Z>-0.855146508)=0.8037650223$.

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

6. Independent measurements follow $N(\mu, 81)$ with known variance 81. Investigators ask whether the population mean exceeds 56. State H0 and HA, classify each as simple or composite, and name the tail direction.

Assumptions: Measurements are iid normal with unknown fixed mean μ and known $\sigma=9$.

Reference: A one-sided null includes its boundary and all parameter values opposite HA.

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

7. A valid test of H0 versus HA reports $p=0.121$ at $\alpha=0.01$. State the decision and write a conclusion that does not treat failure to reject as proof of H0.

Assumptions: The test design and assumptions were checked before the p-value was calculated.

Reference: Reject when $p \leq \alpha$; otherwise fail to reject.

Hypothesis Tests, Errors, and Power

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

8. Test H0: $\mu=58$ against HA: $\mu>58$ with known $\sigma=11$, $n=25$, and $\alpha=0.05$. Find the critical sample mean and power at $\mu=65$.

Assumptions: Observations are independent normal measurements and σ is known.

Reference: Normal reference: $z_{(1-\alpha)}=1.644853492$. At $\mu=65$, the printed cutoff standardizes to $z=-1.536964690$, with $P(Z>-1.536964690)=0.9378490030$.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H_0 and H_A before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

1.

For T in $\{0,1,2,3\}$, H_0 has PMF $(0.44,0.31,0.17,0.08)$ and H_A has PMF $(0.08,0.17,0.27,0.48)$. Choose between rejection sets $\{3\}$ and $\{2,3\}$ when alpha may not exceed 0.090. Report the chosen set and its alpha.

Assumptions: The displayed PMFs are complete and outcomes are mutually exclusive.

Reference: For each candidate, alpha is the sum of its probabilities under H_0 .

Answer: Reject for T in $\{3\}$; alpha=0.08.

1. For $\{3\}$, alpha=0.08. For $\{2,3\}$, alpha=0.25.

2. The selected set has the greater alternative probability among candidates satisfying alpha<=0.090.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H_0 and H_A before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

2. A valid test of H_0 versus H_A reports $p=0.056$ at $\alpha=0.05$. State the decision and write a conclusion that does not treat failure to reject as proof of H_0 .

Assumptions: The test design and assumptions were checked before the p-value was calculated.

Reference: Reject when $p \leq \alpha$; otherwise fail to reject.

Answer: Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

- 1. Compare 0.056 with 0.05.
- 2. The result is inconclusive about H_A ; H_0 has not been proved.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

3. For T in {0,1,2,3}, H0 has PMF (0.46,0.27,0.19,0.08) and HA has PMF (0.10,0.21,0.28,0.41). Reject H0 when T is in {3}. Find alpha, beta, and power.

Assumptions: The displayed hypotheses specify the full sampling distributions.
Reference: $\alpha = P(\text{reject}|H_0)$; $\beta = P(\text{fail to reject}|H_A)$; $\text{power} = 1 - \beta = P(\text{reject}|H_A)$.

Answer: $\alpha=0.08$, $\beta=0.59$, $\text{power}=0.41$.
1. $\alpha=0.08=0.08$.
2. $\text{power}=0.41=0.41$; $\beta=1-\text{power}=0.59$.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

4. Independent measurements follow $N(\mu, 49)$ with known variance 49. Investigators ask whether the population mean exceeds 54. State H0 and HA, classify each as simple or composite, and name the tail direction.

Assumptions: Measurements are iid normal with unknown fixed mean μ and known $\sigma^2=49$.

Reference: A one-sided null includes its boundary and all parameter values opposite HA.

Answer: H0: $\mu \leq 54$ (composite); HA: $\mu > 54$ (composite); upper-tailed.

- 1. The research direction is greater than 54.
- 2. The null contains every $\mu \leq 54$; its boundary $\mu = 54$ supplies the test distribution.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

5. Test H0: $\mu=55$ against HA: $\mu>55$ with known $\sigma=8$, $n=25$, and $\alpha=0.05$. Find the critical sample mean and power at $\mu=59$.

Assumptions: Observations are independent normal measurements and σ is known.
Reference: Normal reference: $z_{(1-\alpha)}=1.644853492$. At $\mu=59$, the printed cutoff standardizes to $z=-0.855146508$, with $P(Z>-0.855146508)=0.8037650223$.

Answer: Reject when $\bar{x}>57.6318$; power= 0.8038 .

- 1. Critical $\bar{x}=55+1.644853492(8/\sqrt{25})=57.631765587$.
- 2. Power= $P_{\mu=59}(\bar{x}>57.631765587)=P(Z>-0.855146508)=0.8037650223$, so the final answer is 0.8038 .

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

6. Independent measurements follow $N(\mu, 81)$ with known variance 81. Investigators ask whether the population mean exceeds 56. State H0 and HA, classify each as simple or composite, and name the tail direction.

Assumptions: Measurements are iid normal with unknown fixed mean μ and known $\sigma^2=9$.

Reference: A one-sided null includes its boundary and all parameter values opposite HA.

Answer: H0: $\mu \leq 56$ (composite); HA: $\mu > 56$ (composite); upper-tailed.

- 1. The research direction is greater than 56.
- 2. The null contains every $\mu \leq 56$; its boundary $\mu = 56$ supplies the test distribution.

Hypothesis Tests, Errors, and Power

Worked Answer Key

State H_0 and H_A before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

7. A valid test of H_0 versus H_A reports $p=0.121$ at $\alpha=0.01$. State the decision and write a conclusion that does not treat failure to reject as proof of H_0 .
- Assumptions: The test design and assumptions were checked before the p-value was calculated.
- Reference: Reject when $p \leq \alpha$; otherwise fail to reject.
- Answer: Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.
1. Compare 0.121 with 0.01.
 2. The result is inconclusive about H_A ; H_0 has not been proved.

Hypothesis Tests, Errors, and Power
Worked Answer Key

State H0 and HA before selecting a rejection rule. Track every probability under the distribution named in its conditioning statement.

8. Test H0: $\mu=58$ against HA: $\mu>58$ with known $\sigma=11$, $n=25$, and $\alpha=0.05$. Find the critical sample mean and power at $\mu=65$.

Assumptions: Observations are independent normal measurements and σ is known.

Reference: Normal reference: $z_{(1-\alpha)}=1.644853492$. At $\mu=65$, the printed cutoff standardizes to $z=-1.536964690$, with $P(Z>-1.536964690)=0.9378490030$.

Answer: Reject when $\bar{x}>61.6187$; power=0.9378.

1. Critical $\bar{x}=58+1.644853492(11/\sqrt{25})=61.618677682$.

2. Power= $P_{\mu=65}(\bar{x}>61.618677682)=P(Z>-1.536964690)=0.9378490030$, so the final answer is 0.9378.