Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

1. Complete the Bayesian update table after observing data D, and find every P(H_i | D).

Given: The listed hypotheses are mutually exclusive and exhaustive. Likelihood means P(D | H_i).

Hypothesis	Prior	Likelihood	Prior x likelihood	Posterior
H1	0.20	0.70	0.1400	?
H2	0.20	0.30	0.0600	?
H3	0.30	0.60	0.1800	?
H4	0.30	0.20	0.0600	?

2. A three-hypothesis prior is (0.00, 0.10, 0.90), and likelihoods of D are (0.80, 0.50, 0.15). Find the posterior and explain H1.

Given: Ordinary Bayes updating multiplies prior by likelihood before normalization.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

3. **A sensor has one of three fixed reliability levels. Its next 4 reports are recorded. Identify the hypothesis, data, prior, likelihood, and posterior roles.**

Given: P(data | hypothesis) and P(hypothesis | data) reverse the conditioning direction.

4. **Two hypotheses give success probabilities and 0.7. Starting from equal priors, observe 0 success(es) and 1 failure(s), for 1 observation(s) total. Update sequentially and verify the batch posterior.**

Given: Observations are conditionally independent given the fixed hypothesis.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

5. **A model has 2 exhaustive hypotheses with prior (0.444, 0.556). Every hypothesis assigns likelihood 0 to observed data D. Can a posterior be formed by ordinary conditioning?**

Given: The evidence is the prior-weighted sum of likelihoods.

6. **A sensor has one of three fixed reliability levels. Its next 7 reports are recorded. Identify the hypothesis, data, prior, likelihood, and posterior roles.**

Given: P(data | hypothesis) and P(hypothesis | data) reverse the conditioning direction.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

7. Complete the Bayesian update table after observing data D, and find every P(H_i | D).

Given: The listed hypotheses are mutually exclusive and exhaustive. Likelihood means P(D | H_i).

Hypothesis	Prior	Likelihood	Prior x likelihood	Posterior
H1	0.20	0.60	0.1200	?
H2	0.20	0.20	0.0400	?
H3	0.30	0.50	0.1500	?
H4	0.30	0.80	0.2400	?

8. Two hypotheses give success probabilities and 0.8. Starting from equal priors, observe 0 success(es) and 2 failure(s), for 2 observation(s) total. Update sequentially and verify the batch posterior.

Given: Observations are conditionally independent given the fixed hypothesis.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Worked Answer Key

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

1. **Complete the Bayesian update table after observing data D, and find every P(H_i | D).**

Hypothesis	Prior	Likelihood	Prior x likelihood	Posterior
H1	0.20	0.70	0.1400	?
H2	0.20	0.30	0.0600	?
H3	0.30	0.60	0.1800	?
H4	0.30	0.20	0.0600	?

Answer: H1: 0.3182, H2: 0.1364, H3: 0.4091, H4: 0.1364

1. Multiply each prior by its likelihood.
2. Evidence P(D)=sum of Bayes numerators=0.4400.
3. Divide every numerator by the evidence to normalize.

2. **A three-hypothesis prior is (0.00, 0.10, 0.90), and likelihoods of D are (0.80, 0.50, 0.15). Find the posterior and explain H1.**

Answer: Posterior = (0.0000, 0.2703, 0.7297); H1 remains 0

1. H1 has Bayes numerator 0 times its likelihood = 0.
2. Positive likelihood cannot revive a hypothesis assigned prior probability zero.
3. Normalize the remaining positive numerators.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Worked Answer Key

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

3. **A sensor has one of three fixed reliability levels. Its next 4 reports are recorded. Identify the hypothesis, data, prior, likelihood, and posterior roles.**

Answer: Hypothesis: the sensor reliability level; data: the 4 recorded reports.

1. Prior: belief about the hypothesis before these data.
2. Likelihood: probability of these data under a fixed hypothesis.
3. Posterior: updated belief about the hypothesis after these data.

4. **Two hypotheses give success probabilities and 0.7. Starting from equal priors, observe 0 success(es) and 1 failure(s), for 1 observation(s) total. Update sequentially and verify the batch posterior.**

Answer: Posterior = (0.6667, 0.3333) by both methods

1. Batch likelihoods are 0.600000 and 0.300000.
2. Normalizing prior x batch likelihood gives (0.6667, 0.3333).
3. Sequential updating gives (0.6667, 0.3333), agreeing up to rounding.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Worked Answer Key

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

5. **A model has 2 exhaustive hypotheses with prior (0.444, 0.556). Every hypothesis assigns likelihood 0 to observed data D. Can a posterior be formed by ordinary conditioning?**

Answer: No; P(D)=0, so conditioning on D is undefined under this model.

1. Every Bayes numerator is 0.
2. Their sum, the evidence P(D), is 0.
3. Do not divide by zero; revisit the model or data record.

6. **A sensor has one of three fixed reliability levels. Its next 7 reports are recorded. Identify the hypothesis, data, prior, likelihood, and posterior roles.**

Answer: Hypothesis: the sensor reliability level; data: the 7 recorded reports.

1. Prior: belief about the hypothesis before these data.
2. Likelihood: probability of these data under a fixed hypothesis.
3. Posterior: updated belief about the hypothesis after these data.

Name _____ Score _____ Teacher _____ Date _____

Discrete Bayesian Updating

Worked Answer Key

Label hypotheses and data before multiplying likelihood by prior. Normalize the Bayes numerators to obtain posterior probabilities. For sequential evidence, use each posterior as the next prior and verify that it matches the batch likelihood.

7. **Complete the Bayesian update table after observing data D, and find every P(H_i | D).**

Hypothesis	Prior	Likelihood	Prior x likelihood	Posterior
H1	0.20	0.60	0.1200	?
H2	0.20	0.20	0.0400	?
H3	0.30	0.50	0.1500	?
H4	0.30	0.80	0.2400	?

Answer: H1: 0.2182, H2: 0.0727, H3: 0.2727, H4: 0.4364

1. Multiply each prior by its likelihood.
2. Evidence P(D)=sum of Bayes numerators=0.5500.
3. Divide every numerator by the evidence to normalize.

8. **Two hypotheses give success probabilities and 0.8. Starting from equal priors, observe 0 success(es) and 2 failure(s), for 2 observation(s) total. Update sequentially and verify the batch posterior.**

Answer: Posterior = (0.9412, 0.0588) by both methods

1. Batch likelihoods are 0.640000 and 0.040000.
2. Normalizing prior x batch likelihood gives (0.9412, 0.0588).
3. Sequential updating gives (0.9412, 0.0588), agreeing up to rounding.