

Credible Intervals and Prior Elicitation

A credible interval summarizes posterior belief about a parameter. Compute mass from the printed PMF or CDF, use equal tail quantiles when requested, and do not interpret posterior coverage as a repeated-sample confidence statement.

1. An expert states prior $q_{10}=0.0400$, $\text{median}=0.2000$, and $q_{90}=0.3600$, so the central 80% interval is $[0.0400,0.3600]$. Choose the compatible parameterized prior: $F_1(\theta)=(\theta/0.4)^1$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.0400,0.2000,0.3600)$; $F_2(\theta)=(\theta/0.4)^2$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.1265,0.2828,0.3795)$; $F_3(\theta)=(\theta/0.4)^3$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.1857,0.3175,0.3862)$.

Given: For candidate CDF $F_k(\theta)=(\theta/u)^k$ on $[0,u]$, the p-quantile is $u \cdot p^{1/k}$.

2. A posterior PMF on 4 support values is shown. Find $P(2 \leq \theta \leq 3)$ and state the corresponding credible interval mass.

Given: Posterior mass is summed over support values inside the closed interval.

theta	1	2	3	4
Posterior	1/6	1/3	1/6	1/3

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3. A posterior CDF is $F(\theta)=\theta^3$ on $[0,1]$. Find the 50% equal-tail credible interval.

Given: An equal-tail interval leaves $(1-\text{coverage})/2$ posterior probability in each tail.

4. A 95% credible interval for θ is $[4, 6]$. Interpret this Bayesian statement and identify an incorrect confidence-style claim.

Given: θ is modeled as uncertain under the posterior distribution.

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5. A posterior on [0,1] has CDF $F(\theta)=\theta^2$. Find $P(0.3 \leq \theta \leq 0.8)$.

Given: For a continuous posterior, interval mass is a CDF difference; endpoint point masses are zero.

6. A posterior PMF on 5 support values is shown. Find $P(3 \leq \theta \leq 4)$ and state the corresponding credible interval mass.

Given: Posterior mass is summed over support values inside the closed interval.

theta	2	3	4	5	6
Posterior	1/7	2/7	1/7	2/7	1/7

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7. A posterior CDF is $F(\theta)=\theta^1$ on $[0,1]$. Find the 90% equal-tail credible interval.

Given: An equal-tail interval leaves $(1-\text{coverage})/2$ posterior probability in each tail.

8. A 90% credible interval for θ is $[2, 4]$. Interpret this Bayesian statement and identify an incorrect confidence-style claim.

Given: θ is modeled as uncertain under the posterior distribution.

Credible Intervals and Prior Elicitation

Worked Answer Key

A credible interval summarizes posterior belief about a parameter. Compute mass from the printed PMF or CDF, use equal tail quantiles when requested, and do not interpret posterior coverage as a repeated-sample confidence statement.

1. An expert states prior $q_{10}=0.0400$, $\text{median}=0.2000$, and $q_{90}=0.3600$, so the central 80% interval is $[0.0400,0.3600]$. Choose the compatible parameterized prior: $F_1(\theta)=(\theta/0.4)^1$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.0400,0.2000,0.3600)$; $F_2(\theta)=(\theta/0.4)^2$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.1265,0.2828,0.3795)$; $F_3(\theta)=(\theta/0.4)^3$ on $[0,0.4]$, with $(q_{10},q_{50},q_{90})=(0.1857,0.3175,0.3862)$.

Answer: F_1

1. For F_1 , $q_p=0.4 p^{(1/1)}$.
2. Its q_{10} , median, and q_{90} are 0.0400, 0.2000, and 0.3600, matching every elicited statement at the printed precision.
3. The other candidates' printed quantiles fail at least one stated value.

2. A posterior PMF on 4 support values is shown. Find $P(2 \leq \theta \leq 3)$ and state the corresponding credible interval mass.

θ	1	2	3	4
Posterior	1/6	1/3	1/6	1/3

Answer: $P(2 \leq \theta \leq 3)=1/2=0.5000=50\%$

1. Include support values from 2 through 3.
2. Add $1/3 + 1/6 = 1/2$.
3. $[2,3]$ is a 50% credible interval for this posterior.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

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Worked Answer Key

A credible interval summarizes posterior belief about a parameter. Compute mass from the printed PMF or CDF, use equal tail quantiles when requested, and do not interpret posterior coverage as a repeated-sample confidence statement.

3. A posterior CDF is $F(\theta) = \theta^3$ on $[0,1]$. Find the 50% equal-tail credible interval.

Answer: [0.6300, 0.9086]

1. Each tail has mass 0.2500.
2. Solve $\theta^3 = 0.2500$ and $\theta^3 = 0.7500$.
3. The endpoints are 0.6300 and 0.9086.

4. A 95% credible interval for θ is $[4, 6]$. Interpret this Bayesian statement and identify an incorrect confidence-style claim.

Answer: Given the model and data, posterior probability 0.95 is assigned to θ in $[4,6]$.

1. The probability statement is about posterior belief in θ after observing the data.
2. It does not say this fixed realized interval captures θ in 90% or 95% of repeated samples.
3. It is not a p-value or a posterior probability of a point null.

Credible Intervals and Prior Elicitation

Worked Answer Key

A credible interval summarizes posterior belief about a parameter. Compute mass from the printed PMF or CDF, use equal tail quantiles when requested, and do not interpret posterior coverage as a repeated-sample confidence statement.

5. A posterior on [0,1] has CDF $F(\theta)=\theta^2$. Find $P(0.3 \leq \theta \leq 0.8)$.

Answer: 0.5500

- 1. $P=F(0.8)-F(0.3)$.
- 2. $=0.8^2-0.3^2$.
- 3. The posterior interval mass is 0.5500.

6. A posterior PMF on 5 support values is shown. Find $P(3 \leq \theta \leq 4)$ and state the corresponding credible interval mass.

theta	2	3	4	5	6
Posterior	1/7	2/7	1/7	2/7	1/7

Answer: $P(3 \leq \theta \leq 4)=3/7=0.4286=42.9\%$

- 1. Include support values from 3 through 4.
- 2. Add $2/7 + 1/7 = 3/7$.
- 3. [3,4] is a 42.9% credible interval for this posterior.

Math Sheet

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Credible Intervals and Prior Elicitation

Worked Answer Key

A credible interval summarizes posterior belief about a parameter. Compute mass from the printed PMF or CDF, use equal tail quantiles when requested, and do not interpret posterior coverage as a repeated-sample confidence statement.

7. A posterior CDF is $F(\theta) = \theta^4$ on $[0,1]$. Find the 90% equal-tail credible interval.

Answer: [0.0500, 0.9500]

1. Each tail has mass 0.0500.
2. Solve $\theta^4 = 0.0500$ and $\theta^4 = 0.9500$.
3. The endpoints are 0.0500 and 0.9500.

8. A 90% credible interval for θ is $[2, 4]$. Interpret this Bayesian statement and identify an incorrect confidence-style claim.

Answer: Given the model and data, posterior probability 0.9 is assigned to θ in $[2,4]$.

1. The probability statement is about posterior belief in θ after observing the data.
2. It does not say this fixed realized interval captures θ in 90% or 95% of repeated samples.
3. It is not a p-value or a posterior probability of a point null.