

Continuous-Prior Bayesian Updating

The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

1. Prior density $f(\theta)=3 \theta^2$ on $[0,1]$. Ordered Bernoulli data contain 3 success(es) and 1 failure(s). Find the normalized posterior density.

Given: The ordered-data likelihood is $\theta^s(1-\theta)^f$; θ is a continuous hypothesis in $[0,1]$.

2. Prior density $f(\theta)=1$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). Find the posterior predictive probability that the next trial succeeds.

Given: Prediction averages θ over the posterior density.

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The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

- 3. Prior density $f(\theta)=2\theta$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). Compute the evidence $P(\text{data})$ by integration.**

Given: The observations are an ordered sequence, so no binomial coefficient is included.

- 4. Prior density $f(\theta)=3\theta^2$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). State the parameter support and find $P(\theta=0.75)$. Explain why the posterior density height at 0.75 is not that probability.**

Given: θ has a continuous prior and posterior density.

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The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

5. Prior density $f(\theta)=1$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). Find the normalized posterior density.

Given: The ordered-data likelihood is $\theta^2(1-\theta)^0$; θ is a continuous hypothesis in $[0,1]$.

6. Prior density $f(\theta)=2\theta$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). Compute the evidence $P(\text{data})$ by integration.

Given: The observations are an ordered sequence, so no binomial coefficient is included.

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The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

7. Prior density $f(\theta)=3 \theta^2$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). State the parameter support and find $P(\theta=0.75)$. Explain why the posterior density height at 0.75 is not that probability.

Given: θ has a continuous prior and posterior density.

8. Prior density $f(\theta)=1$ on $[0,1]$. Ordered Bernoulli data contain 3 success(es) and 0 failure(s). Find the posterior predictive probability that the next trial succeeds.

Given: Prediction averages θ over the posterior density.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Continuous-Prior Bayesian Updating

Worked Answer Key

The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

1. Prior density $f(\theta) = 3\theta^2$ on $[0,1]$. Ordered Bernoulli data contain 3 success(es) and 1 failure(s). Find the normalized posterior density.

Answer: $f(\theta|\text{data}) = 42\theta^5(1-\theta)^1$ on $[0,1]$

1. Prior x likelihood is proportional to $\theta^5(1-\theta)^1$.
2. This is $\text{Beta}(6,2)$; its normalizing constant is 42.
3. The density is 0 outside $[0,1]$.

2. Prior density $f(\theta) = 1$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). Find the posterior predictive probability that the next trial succeeds.

Answer: $P(\text{next success}|\text{data}) = 2/3 = 0.6667$

1. The posterior is $\text{Beta}(2,1)$.
2. Its mean is $a/(a+b)$.
3. The predictive success probability is $2/3$.

Continuous-Prior Bayesian Updating

Worked Answer Key

The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

3. Prior density $f(\theta)=2\theta$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). Compute the evidence $P(\text{data})$ by integration.

Answer: $P(\text{data})=2/3=0.6667$

1. $P(\text{data})=\int_0^1 2\theta(1-\theta)^0 d\theta$.

2. Use $\int_0^1 \theta^{(a-1)}(1-\theta)^{(b-1)} d\theta = \frac{(a-1)!(b-1)!}{(a+b-1)!}$.

3. The result is $2/3$.

4. Prior density $f(\theta)=3\theta^2$ on $[0,1]$. Ordered Bernoulli data contain 1 success(es) and 0 failure(s). State the parameter support and find $P(\theta=0.75)$. Explain why the posterior density height at 0.75 is not that probability.

Answer: Support $[0,1]$; $P(\theta=0.75)=0$

1. A continuous parameter assigns probability by integrating density over intervals.

2. The single point 0.75 has width 0, so its probability is 0.

3. A density height can exceed 1 and is not a point mass.

Math Sheet

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Continuous-Prior Bayesian Updating

Worked Answer Key

The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

5. Prior density $f(\theta)=1$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). Find the normalized posterior density.

Answer: $f(\theta|\text{data})=3\theta^2(1-\theta)^0$ on $[0,1]$

1. Prior $\times$ likelihood is proportional to $\theta^2(1-\theta)^0$.
2. This is $\text{Beta}(3,1)$; its normalizing constant is 3.
3. The density is 0 outside $[0,1]$.

6. Prior density $f(\theta)=2\theta$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). Compute the evidence $P(\text{data})$ by integration.

Answer: $P(\text{data})=1/2=0.5000$

1. $P(\text{data})=\int_0^1 2\theta^3(1-\theta)^0 d\theta$.
2. Use $\int_0^1 \theta^a(1-\theta)^b = \frac{a!(b-1)!}{(a+b-1)!}$.
3. The result is $1/2$.

Math Sheet

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Continuous-Prior Bayesian Updating

Worked Answer Key

The parameter θ has support $[0,1]$. Multiply the prior density by the likelihood, integrate to find the evidence, and normalize to a posterior density. A density height is not probability at one point.

7. Prior density $f(\theta)=3\theta^2$ on $[0,1]$. Ordered Bernoulli data contain 2 success(es) and 0 failure(s). State the parameter support and find $P(\theta=0.75)$. Explain why the posterior density height at 0.75 is not that probability.

Answer: Support $[0,1]$; $P(\theta=0.75)=0$

1. A continuous parameter assigns probability by integrating density over intervals.
2. The single point 0.75 has width 0, so its probability is 0.
3. A density height can exceed 1 and is not a point mass.

8. Prior density $f(\theta)=1$ on $[0,1]$. Ordered Bernoulli data contain 3 success(es) and 0 failure(s). Find the posterior predictive probability that the next trial succeeds.

Answer: $P(\text{next success}|\text{data})=4/5=0.8000$

1. The posterior is $\text{Beta}(4,1)$.
2. Its mean is $a/(a+b)$.
3. The predictive success probability is $4/5$.