

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

1. Observed counts are (0,8,14,10) across 4 categories. H_0 assigns equal probabilities. Use likelihood-ratio G at $\alpha=0.05$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (8, 8, 8, 8). Use 0 $\log(0/E)=0$ in $G=2 \sum O \log(O/E)$. $df=k-1-0=3$; Chi-square reference: $P(\chi^2_3 \geq 20.132113088)=0.0001593703$. Critical=7.814727903.

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

2. Observed 2x3 table rows are [13,18,10] and [20,15,16]. Test independence using likelihood-ratio G at $\alpha=0.1$.

Assumptions: Observations are independent counts; categories are mutually exclusive; verify expected counts.

Reference: Expected $E_{ij}=(\text{row total})(\text{column total})/N$. $G=2 \sum O \log(O/E)$, with $0 \log(0/E)=0$. $df=(r-1)(c-1)=2$; Chi-square reference: $P(\chi^2_{2} \geq 2.077360604)=0.3539214433$. Critical= 4.605170186 .

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 log(0/E) as 0 and check count adequacy.

3. Observed 2x2 table rows are [17,10] and [8,19]. Test independence using Pearson χ^2 at $\alpha=0.01$.

Assumptions: Observations are independent counts; categories are mutually exclusive; verify expected counts.

Reference: Expected $E_{ij}=(\text{row total})(\text{column total})/N$. $\chi^2=\sum (O-E)^2/E$. $df=(r-1)(c-1)=1$; Chi-square reference: $P(\chi^2_{1}\geq 6.033103448)=0.0140400271$. Critical=6.634896601.

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

4. Observed counts are (8,11,14) across 3 categories. H_0 assigns equal probabilities. Use Pearson χ^2 at $\alpha=0.05$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (11, 11, 11). Use $\chi^2 = \sum (O-E)^2/E$. $df=k-1-0=2$; Chi-square reference: $P(\chi^2_{2} \geq 1.636363636) = 0.4412331678$. Critical = 5.991464547.

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson X2 or likelihood-ratio G. Treat 0 log(0/E) as 0 and check count adequacy.

5. Observed counts are (2,5,6) across 3 categories. H0 assigns equal probabilities. Use Pearson X2 at alpha=0.1 and decide whether the chi-square approximation is adequate.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (4.333, 4.333, 4.333). Use $X^2 = \sum (O - E)^2 / E$. $df = k - 1 - 0 = 2$; Chi-square reference: $P(\text{ChiSq}_2 \geq 2.000000000) = 0.3678794412$. Critical = 4.605170186.

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

6. Observed counts are (6,9,9,9,12) across 5 categories. H_0 has $P(C_1)=\dots=P(C_4)=p$ and $P(C_5)=1-4p$; estimate p from these data. Use Pearson χ^2 at $\alpha=0.01$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (8.25, 8.25, 8.25, 8.25, 12). The MLE is $\hat{p}=\text{sum of the first 4 counts}/[4N]=0.183333333$. Use $\chi^2=\text{sum } (O-E)^2/E$. $df=k-1-1=3$; Chi-square reference: $P(\chi^2_{3}>0.818181818)=0.8451134217$. Critical= 11.344866730 .

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson X2 or likelihood-ratio G. Treat 0 log(0/E) as 0 and check count adequacy.

7. Observed counts are (1,3,3,3,5) across 5 categories. H0 assigns equal probabilities. Use Pearson X2 at alpha=0.05 and decide whether the chi-square approximation is adequate.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (3, 3, 3, 3, 3). Use $X^2 = \sum (O-E)^2/E$. $df = k-1 = 0 = 4$; Chi-square reference: $P(\text{ChiSq}_4 \geq 2.666666667) = 0.6150599889$. Critical = 9.487729037.

Chi-Square Goodness-of-Fit and Independence Tests

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

8. Observed counts are (8,11,14) across 3 categories. H_0 assigns equal probabilities. Use likelihood-ratio G at $\alpha=0.1$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (11, 11, 11). Use 0 $\log(0/E)=0$ in $G=2 \sum O \log(O/E)$. $df=k-1=0=2$; Chi-square reference: $P(\chi^2_{2} \geq 1.657277893)=0.4366431768$. Critical=4.605170186.

Name _____ Score _____ Teacher _____ Date _____

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

1. Observed counts are (0,8,14,10) across 4 categories. H_0 assigns equal probabilities. Use likelihood-ratio G at $\alpha=0.05$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (8, 8, 8, 8). Use 0 $\log(0/E)=0$ in $G=2 \sum O \log(O/E)$. $df=k-1-0=3$; Chi-square reference: $P(\chi^2_{3} \geq 20.132113088)=0.0001593703$. Critical=7.814727903.

Answer: statistic=20.1321, $df=3$, $p=0.0002$; adequacy=satisfied; Reject H_0 at $\alpha=0.05$; the data support H_A .

1. 4 expected counts are 8, 8, 8, 8.

2. $G=20.132113088$; $df=4-1-0=3$; computed tail $p=0.0001593703$.

3. All expected counts are at least 5. Reject H_0 at $\alpha=0.05$; the data support H_A .

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(O/E)$ as 0 and check count adequacy.

2. Observed 2x3 table rows are [13,18,10] and [20,15,16]. Test independence using likelihood-ratio G at $\alpha=0.1$.

Assumptions: Observations are independent counts; categories are mutually exclusive; verify expected counts.
Reference: Expected $E_{ij}=(\text{row total})(\text{column total})/N$. $G=2 \sum O \log(O/E)$, with 0 $\log(O/E)=0$. $df=(r-1)(c-1)=2$; Chi-square reference: $P(\text{ChiSq}_2 \geq 2.077360604)=0.3539214433$. Critical= 4.605170186 .
Answer: statistic=2.0774, df=2, p=0.3539; expected counts are adequate; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.
1. Row totals=41,51; column totals=33,33,26; expected rows [14.707, 14.707, 11.587] and [18.293, 18.293, 14.413].
2. $G=2.077360604$; use printed chi-square survival 0.3539214433.
3. All expected counts at least 5: yes. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 log(0/E) as 0 and check count adequacy.

3. Observed 2x2 table rows are [17,10] and [8,19]. Test independence using Pearson χ^2 at $\alpha=0.01$.

Assumptions: Observations are independent counts; categories are mutually exclusive; verify expected counts.

Reference: Expected $E_{ij}=(\text{row total})(\text{column total})/N$. $\chi^2=\sum (O-E)^2/E$. $df=(r-1)(c-1)=1$; Chi-square reference: $P(\chi^2_{1,0.01}\geq 6.633103448)=0.0140400271$. Critical=6.634896601.

Answer: statistic=6.0331, df=1, p=0.014; expected counts are adequate; Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

- 1. Row totals=27,27; column totals=25,29; expected rows [12.5, 14.5] and [12.5, 14.5].
- 2. $\chi^2=6.033103448$; use printed chi-square survival 0.0140400271.
- 3. All expected counts at least 5: yes. Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(O/E)$ as 0 and check count adequacy.

4. Observed counts are (8,11,14) across 3 categories. H_0 assigns equal probabilities. Use Pearson χ^2 at $\alpha=0.05$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (11, 11, 11). Use $\chi^2 = \sum (O-E)^2/E$. $df=k-1=0=2$; Chi-square reference: $P(\chi^2_{2} \geq 1.636363636) = 0.4412331678$. Critical = 5.991464547.

Answer: statistic=1.6364, $df=2$, $p=0.4412$; adequacy=satisfied; Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

- 3 expected counts are 11, 11, 11.
- $\chi^2=1.636363636$; $df=3-1=0=2$; computed tail $p=0.4412331678$.
- All expected counts are at least 5. Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(O/E)$ as 0 and check count adequacy.

5. Observed counts are (2,5,6) across 3 categories. H_0 assigns equal probabilities. Use Pearson χ^2 at $\alpha=0.1$ and decide whether the chi-square approximation is adequate.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (4.333, 4.333, 4.333). Use $\chi^2 = \sum (O-E)^2/E$. $df=k-1-0=2$; Chi-square reference: $P(\chi^2_{2} \geq 2.000000000) = 0.3678794412$. Critical = 4.605170186.

Answer: statistic=2.0000, $df=2$, $p=0.3679$; adequacy=not satisfied; The computed asymptotic tail $p=0.3678794412$ is unreliable because an expected count is below 5; make no formal reject/fail-to-reject decision. Consider an exact test or pool categories only when scientifically justified.

1. 3 expected counts are 4.333, 4.333, 4.333.

2. $\chi^2=2.000000000$; $df=3-1-0=2$; computed tail $p=0.3678794412$.

3. At least one expected count is below 5. The chi-square approximation is unreliable, so do not issue a formal decision; use an exact method or scientifically justified pooling.

Name _____ Score _____ Teacher _____ Date _____

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson X^2 or likelihood-ratio G . Treat 0 $\log(0/E)$ as 0 and check count adequacy.

6. Observed counts are (6,9,9,9,12) across 5 categories. H_0 has $P(C_1)=\dots=P(C_4)=p$ and $P(C_5)=1-4p$; estimate p from these data. Use Pearson X^2 at $\alpha=0.01$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (8.25, 8.25, 8.25, 8.25, 12). The MLE is $\hat{p}=\text{sum of the first 4 counts}/[4N]=0.183333333$. Use $X^2=\text{sum } (O-E)^2/E$. $df=k-1-1=3$; Chi-square reference: $P(\text{ChiSq}_3 > 0.818181818)=0.8451134217$. Critical= 11.344866730 .

Answer: statistic=0.8182, df=3, p=0.8451; adequacy=satisfied; Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

1. 5 expected counts are 8.25, 8.25, 8.25, 8.25, 12. The MLE is $\hat{p}=\text{sum of the first 4 counts}/[4N]=0.183333333$.
2. $X^2=0.818181818$; $df=5-1-1=3$; computed tail $p=0.8451134217$.
3. All expected counts are at least 5. Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(O/E)$ as 0 and check count adequacy.

7. Observed counts are (1,3,3,3,5) across 5 categories. H_0 assigns equal probabilities. Use Pearson χ^2 at $\alpha=0.05$ and decide whether the chi-square approximation is adequate.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (3, 3, 3, 3, 3). Use $\chi^2 = \sum (O-E)^2/E$. $df=k-1=4$; Chi-square reference: $P(\chi^2_{4} \geq 2.666666667) = 0.6150599889$. Critical = 9.487729037.

Answer: statistic=2.6667, $df=4$, $p=0.6151$; adequacy=not satisfied; The computed asymptotic tail $p=0.6150599889$ is unreliable because an expected count is below 5; make no formal reject/fail-to-reject decision. Consider an exact test or pool categories only when scientifically justified.

1. 5 expected counts are 3, 3, 3, 3, 3.

2. $\chi^2=2.666666667$; $df=5-1=4$; computed tail $p=0.6150599889$.

3. At least one expected count is below 5. The chi-square approximation is unreliable, so do not issue a formal decision; use an exact method or scientifically justified pooling.

Chi-Square Goodness-of-Fit and Independence Tests

Worked Answer Key

Build expected counts before calculating Pearson χ^2 or likelihood-ratio G . Treat 0 $\log(O/E)$ as 0 and check count adequacy.

8. Observed counts are (8,11,14) across 3 categories. H_0 assigns equal probabilities. Use likelihood-ratio G at $\alpha=0.1$.

Assumptions: Counts are independent and categories are mutually exclusive; expected-count adequacy must be checked.

Reference: Expected counts are (11, 11, 11). Use 0 $\log(O/E)=0$ in $G=2 \sum O \log(O/E)$. $df=k-1=0=2$; Chi-square reference: $P(\chi^2_{2} \geq 1.657277893)=0.4366431768$. Critical=4.605170186.

Answer: statistic=1.6573, $df=2$, $p=0.4366$; adequacy=satisfied; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

1. 3 expected counts are 11, 11, 11.

2. $G=1.657277893$; $df=3-1=2$; computed tail $p=0.4366431768$.

3. All expected counts are at least 5. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.