

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

1. Under H_0 , X is Binomial($n=20$, $p=0.4$); observe $x=10$. Test H_A : p is not 0.4 at $\alpha=0.1$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 20 independent binary trials with one common success probability.
Reference: Exact null table $P_0(X=x)$, $x=0,\dots,20$: 0.0000365616, 0.0004874878, 0.0030874227, 0.0123496907, 0.0349907904, 0.0746470195, 0.1244116992, 0.1658822656, 0.1797057878, 0.1597384780, 0.1171415505, 0.0709948791, 0.0354974396, 0.0145630521, 0.0048543507, 0.0012944935, 0.0002696862, 0.0000423037, 0.0000047004, 0.0000003299, 0.0000000110. Two-sided extremeness includes exactly outcomes with $P_0(X=x) \leq P_0(X=\text{observed})$, using unrounded probabilities for ordering.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

2. Data are 22, 23, 24, 25, 27. With known sigma=4, test H0: mu=21 against HA: mu>mu0 at alpha=0.01.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population sigma is known.

Reference: Normal reference at printed z=1.788854382: P(Z>=|z|)=0.0368190849. Critical z=2.326347015.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

3. Under H_0 , X is Binomial($n=11$, $p=0.6$); observe $x=4$. Test H_A : $p<0.6$ at $\alpha=0.05$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 11 independent binary trials with one common success probability.
Reference: Exact null table $P_0(X=x)$, $x=0,\dots,11$: 0.0000419430, 0.0006920602, 0.0051904512, 0.0233570304, 0.0700710912, 0.1471492915, 0.2207239373, 0.2364899328, 0.1773674496, 0.0886837248, 0.0266051174, 0.0036279706. Use the directed tail including the observed count.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

4. A sample has $\bar{x}=25.1$, $n=16$. With known $\sigma=6$, test $H_0: \mu=23$ against $H_A: \mu<\mu_0$ at $\alpha=0.1$.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population sigma is known.

Reference: Normal reference at printed $z=1.400000000$: $\Phi(z)=0.9192432862$. Critical $z=-1.281551956$.

Exact Binomial and Known-Sigma z Tests

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5. Under H_0 , X is Binomial($n=13$, $p=0.4$); observe $x=8$. Test H_A : $p>0.4$ at $\alpha=0.01$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 13 independent binary trials with one common success probability.

Reference: Exact null table $P_0(X=x)$, $x=0,\dots,13$: 0.0013060694, 0.0113192681, 0.0452770726, 0.1106772886, 0.1844621476, 0.2213545771, 0.1967596241, 0.1311730827, 0.0655865414, 0.0242913116, 0.0064776831, 0.0011777606, 0.0001308623, 0.0000067109. Use the directed tail including the observed count.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

6. Under H_0 , X is Binomial($n=14$, $p=0.5$); observe $x=10$. Test H_A : $p>0.5$ at $\alpha=0.05$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 14 independent binary trials with one common success probability.
Reference: Exact null table $P_0(X=x)$, $x=0,\dots,14$: 0.0000610352, 0.0008544922, 0.0055541992, 0.0222167969, 0.0610961914, 0.1221923828, 0.1832885742, 0.2094726563, 0.1832885742, 0.1221923828, 0.0610961914, 0.0222167969, 0.0055541992, 0.0008544922, 0.0000610352. Use the directed tail including the observed count.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

7. A sample has $\bar{x}=23.7$, $n=16$. With known $\sigma=9$, test $H_0: \mu=26$ against $H_A: \mu<\mu_0$ at $\alpha=0.1$.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population sigma is known.

Reference: Normal reference at printed $z=-1.022222222$: $\Phi(z)=0.1533378861$. Critical $z=-1.281551956$.

Exact Binomial and Known-Sigma z Tests

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

8. Under H_0 , X is Binomial($n=16$, $p=0.3$); observe $x=7$. Test H_A : p is not 0.3 at $\alpha=0.01$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 16 independent binary trials with one common success probability.
Reference: Exact null table $P_0(X=x)$, $x=0,\dots,16$: 0.0033232931, 0.0227882952, 0.0732480919, 0.1464961837, 0.2040482559, 0.2098782061, 0.1649043048, 0.1009618193, 0.0486780200, 0.0185440076, 0.0055632023, 0.0013004888, 0.0002322302, 0.0000306238, 0.0000028124, 0.0000001607, 0.0000000043. Two-sided extremeness includes exactly outcomes with $P_0(X=x) \leq P_0(X=\text{observed})$, using unrounded probabilities for ordering.

Exact Binomial and Known-Sigma z Tests

Worked Answer Key

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

1.

Under H_0 , X is Binomial($n=20$, $p=0.4$); observe $x=10$. Test H_A : p is not 0.4 at $\alpha=0.1$. Compute the exact p-value, critical region, and conclusion.

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Answer: $p=0.3703$; critical region $\{0,1,2,3,4,13,14,15,16,17,18,19,20\}$; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

- Included outcomes: $\{0,1,2,3,4,5,10,11,12,13,14,15,16,17,18,19,20\}$.
- Sum their printed ten-decimal table entries: $p=0.3702617694$, which rounds to 0.3703.
- The $\alpha=0.1$ critical region is $\{0,1,2,3,4,13,14,15,16,17,18,19,20\}$. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

Exact Binomial and Known-Sigma z Tests

Worked Answer Key

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

2. Data are 22, 23, 24, 25, 27. With known sigma=4, test H0: mu=21 against HA: mu>mu0 at alpha=0.01.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population sigma is known.

Reference: Normal reference at printed z=1.788854382: P(Z>=|z|)=0.0368190849. Critical z=2.326347015.

Answer: z=1.7889, p=0.0368; critical region z>2.326347015; Fail to reject H0 at alpha=0.01; the data do not establish that H0 is true.

1. $\bar{x}=24.2$ from the printed data.

2. $z=(24.2-21)/(4/\sqrt{5})=1.788854382$.

3. Use the printed normal reference to get p=0.0368190849. Reject for $z>2.326347015$. Fail to reject H0 at alpha=0.01; the data do not establish that H0 is true.

Exact Binomial and Known-Sigma z Tests

Worked Answer Key

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

3. Under H_0 , X is Binomial($n=11$, $p=0.6$); observe $x=4$. Test H_A : $p<0.6$ at $\alpha=0.05$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 11 independent binary trials with one common success probability.

Reference: Exact null table $P_0(X=x)$, $x=0,\dots,11$: 0.0000419430, 0.0006920602, 0.0051904512, 0.0233570304, 0.0700710912, 0.1471492915, 0.2207239373, 0.2364899328, 0.1773674496, 0.0886837248, 0.0266051174, 0.0036279706. Use the directed tail including the observed count.

Answer: $p=0.0994$; critical region $\{0,1,2,3\}$; Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

- 1. Included outcomes: $\{0,1,2,3,4\}$.
- 2. Sum their printed ten-decimal table entries: $p=0.0993525760$, which rounds to 0.0994.
- 3. The $\alpha=0.05$ critical region is $\{0,1,2,3\}$. Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

Exact Binomial and Known-Sigma z Tests

Worked Answer Key

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

4. A sample has $\bar{x}=25.1$, $n=16$. With known $\sigma=6$, test $H_0: \mu=23$ against $H_A: \mu<\mu_0$ at $\alpha=0.1$.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population σ is known.

Reference: Normal reference at printed $z=1.400000000$: $\Phi(z)=0.9192432862$. Critical $z=-1.281551956$.

Answer: $z=1.4000$, $p=0.9192$; critical region $z<-1.281551956$; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

- 1. Use the supplied summary.
- 2. $z=(25.1-23)/(6/\sqrt{16})=1.400000000$.
- 3. Use the printed normal reference to get $p=0.9192432862$. Reject for $z<-1.281551956$. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

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Worked Answer Key

For exact two-sided tests, include outcomes whose null probability is no larger than the observed outcome. For z tests, sigma is known.

5. Under H_0 , X is Binomial($n=13$, $p=0.4$); observe $x=8$. Test H_A : $p>0.4$ at $\alpha=0.01$. Compute the exact p-value, critical region, and conclusion.

Assumptions: There are 13 independent binary trials with one common success probability.

Reference: Exact null table $P_0(X=x)$, $x=0,\dots,13$: 0.0013060694, 0.0113192681, 0.0452770726, 0.1106772886, 0.1844621476, 0.2213545771, 0.1967596241, 0.1311730827, 0.0655865414, 0.0242913116, 0.0064776831, 0.0011777606, 0.0001308623, 0.0000067109. Use the directed tail including the observed count.

Answer: $p=0.0977$; critical region $\{10,11,12,13\}$; Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

- Included outcomes: $\{8,9,10,11,12,13\}$.
- Sum their printed ten-decimal table entries: $p=0.0976708699$, which rounds to 0.0977.
- The $\alpha=0.01$ critical region is $\{10,11,12,13\}$. Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

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Worked Answer Key

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Assumptions: There are 14 independent binary trials with one common success probability.

Reference: Exact null table $P_0(X=x)$, $x=0,\dots,14$: 0.0000610352, 0.0008544922, 0.0055541992, 0.0222167969, 0.0610961914, 0.1221923828, 0.1832885742, 0.2094726563, 0.1832885742, 0.1221923828, 0.0610961914, 0.0222167969, 0.0055541992, 0.0008544922, 0.0000610352. Use the directed tail including the observed count.

Answer: $p=0.0898$; critical region $\{11,12,13,14\}$; Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

- Included outcomes: $\{10,11,12,13,14\}$.
- Sum their printed ten-decimal table entries: $p=0.0897827149$, which rounds to 0.0898.
- The $\alpha=0.05$ critical region is $\{11,12,13,14\}$. Fail to reject H_0 at $\alpha=0.05$; the data do not establish that H_0 is true.

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7. A sample has $\bar{x}=23.7$, $n=16$. With known $\sigma=9$, test $H_0: \mu=26$ against $H_A: \mu<\mu_0$ at $\alpha=0.1$.

Assumptions: The observations are independent and normal, or n is adequate for the mean; population σ is known.

Reference: Normal reference at printed $z=-1.02222222$: $\Phi(z)=0.1533378861$. Critical $z=-1.281551956$.

Answer: $z=-1.0222$, $p=0.1533$; critical region $z<-1.281551956$; Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

- Use the supplied summary.
- $z=(23.7-26)/(9/\sqrt{16})=-1.02222222$.
- Use the printed normal reference to get $p=0.1533378861$. Reject for $z<-1.281551956$. Fail to reject H_0 at $\alpha=0.1$; the data do not establish that H_0 is true.

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Answer: $p=0.2747$; critical region $\{0,11,12,13,14,15,16\}$; Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.

- Included outcomes: $\{0,1,2,7,8,9,10,11,12,13,14,15,16\}$.
- Sum their printed ten-decimal table entries: $p=0.2746730496$, which rounds to 0.2747.
- The $\alpha=0.01$ critical region is $\{0,11,12,13,14,15,16\}$. Fail to reject H_0 at $\alpha=0.01$; the data do not establish that H_0 is true.