

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Beta Conjugate Priors

Use positive integer Beta shapes. Interpret $a+b$ as prior strength and $a/(a+b)$ as the mean. For geometric data, F is the number of failures before the first success, so observing $F=f$ contributes one success and f failures.

1. Start with Beta(2,6). Observe one Bernoulli outcome success. Find the posterior.

Given: The likelihood contributes one θ factor per success and one $(1-\theta)$ factor per failure.

2. Find c so $f(\theta)=c \theta^2(1-\theta)^5$ is a PDF on $[0,1]$. Identify its Beta family.

Given: Shapes a and b are positive integers.

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3. For a Beta(4,6) prior, find its mean and prior strength, and describe whether it favors θ below, near, or above $1/2$.

Given: Use mean $a/(a+b)$ and strength $a+b$.

4. A Beta(5,6) prior is updated by 3 successes and 1 failures. Compare prior and posterior means and give the one-step predictive success probability.

Given: For a Bernoulli parameter, the posterior mean is the next-success predictive probability.

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5. Find c so $f(\theta)=c \theta^5(1-\theta)^5$ is a PDF on $[0,1]$. Identify its Beta family.

Given: Shapes a and b are positive integers.

6. For a $\text{Beta}(1,1)$ prior, find its mean and prior strength, and describe whether it favors θ below, near, or above $1/2$.

Given: Use mean $a/(a+b)$ and strength $a+b$.

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7. A Beta(2,1) prior is updated by 2 successes and 4 failures. Compare prior and posterior means and give the one-step predictive success probability.

Given: For a Bernoulli parameter, the posterior mean is the next-success predictive probability.

8. Start with Beta(3,1). Observe one Bernoulli outcome failure. Find the posterior.

Given: The likelihood contributes one θ factor per success and one $(1-\theta)$ factor per failure.

Beta Conjugate Priors

Worked Answer Key

Use positive integer Beta shapes. Interpret $a+b$ as prior strength and $a/(a+b)$ as the mean. For geometric data, F is the number of failures before the first success, so observing $F=f$ contributes one success and f failures.

1. Start with $\text{Beta}(2,6)$. Observe one Bernoulli outcome success. Find the posterior.

Answer: $\text{Beta}(3,6)$

- 1. Add successes to a and failures to b .
- 2. $(2,6)+(1,0)=(3,6)$.
- 3. This is conjugate updating.

2. Find c so $f(\theta)=c \theta^2(1-\theta)^5$ is a PDF on $[0,1]$. Identify its Beta family.

Answer: $c=168$; $\text{Beta}(3,6)$

- 1. $c=(a+b-1)!/[(a-1)!(b-1)!]$.
- 2. $c=8!/[2!5!]=168$.
- 3. The normalized density is $\text{Beta}(3,6)$.

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Beta Conjugate Priors

Worked Answer Key

Use positive integer Beta shapes. Interpret $a+b$ as prior strength and $a/(a+b)$ as the mean. For geometric data, F is the number of failures before the first success, so observing $F=f$ contributes one success and f failures.

3. For a Beta(4,6) prior, find its mean and prior strength, and describe whether it favors theta below, near, or above 1/2.

Answer: Mean $2/5=0.4000$; strength 10; favors smaller theta

1. Mean= $4/10$.
2. Strength= $4+6=10$.
3. Comparing a and b describes the direction of prior concentration.

4. A Beta(5,6) prior is updated by 3 successes and 1 failures. Compare prior and posterior means and give the one-step predictive success probability.

Answer: Prior mean 0.4545; posterior/predictive mean 0.5333

1. Posterior=Beta(8,7).
2. Prior mean= $5/11=0.4545$.
3. Posterior mean and next-success prediction= $8/15=0.5333$.

Beta Conjugate Priors

Worked Answer Key

Use positive integer Beta shapes. Interpret $a+b$ as prior strength and $a/(a+b)$ as the mean. For geometric data, F is the number of failures before the first success, so observing $F=f$ contributes one success and f failures.

5. Find c so $f(\theta)=c \theta^5(1-\theta)^5$ is a PDF on $[0,1]$. Identify its Beta family.

Answer: $c=2772$; Beta(6,6)

- 1. $c=(a+b-1)!/[(a-1)!(b-1)!]$.
- 2. $c=11!/[(5!5!)=2772]$.
- 3. The normalized density is Beta(6,6).

6. For a Beta(1,1) prior, find its mean and prior strength, and describe whether it favors θ below, near, or above $1/2$.

Answer: Mean $1/2=0.5000$; strength 2; symmetric around $1/2$

- 1. Mean= $1/2$.
- 2. Strength= $1+1=2$.
- 3. Comparing a and b describes the direction of prior concentration.

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Worked Answer Key

Use positive integer Beta shapes. Interpret $a+b$ as prior strength and $a/(a+b)$ as the mean. For geometric data, F is the number of failures before the first success, so observing $F=f$ contributes one success and f failures.

7. A Beta(2,1) prior is updated by 2 successes and 4 failures. Compare prior and posterior means and give the one-step predictive success probability.

Answer: Prior mean 0.6667; posterior/predictive mean 0.4444

1. Posterior=Beta(4,5).
2. Prior mean= $2/3=0.6667$.
3. Posterior mean and next-success prediction= $4/9=0.4444$.

8. Start with Beta(3,1). Observe one Bernoulli outcome failure. Find the posterior.

Answer: Beta(3,2)

1. Add successes to a and failures to b .
2. $(3,1)+(0,1)=(3,2)$.
3. This is conjugate updating.