Vector Formulas

Components, magnitude and direction

From A to B	$\mathbf{v = B - A}$
Length in 3D	$\mathbf{\lvert v\rvert = \sqrt{(v_x^2 + v_y^2 + v_z^2)}}$
Unit direction, v != 0	$\mathbf{u = v/\lvert v\rvert}$
Vector sum	**add matching components**

In 2D omit the z component. Multiplying by a negative scalar reverses direction.

Dot product and angle

Component product sum

$$\mathbf{u \cdot v = u_x v_x + u_y v_y + u_z v_z}$$

Angle for nonzero vectors	$\mathbf{\cos t = (u \cdot v)/(\lvert u\rvert\lvert v\rvert)}$
Perpendicular test	$\mathbf{u \cdot v = 0}$

The perpendicular test describes an angle only when both vectors are nonzero.

Cross product in three dimensions

First component	$\mathbf{u_y v_z - u_z v_y}$
Second component	$\mathbf{u_z v_x - u_x v_z}$
Third component	$\mathbf{u_x v_y - u_y v_x}$
Parallelogram area	$\mathbf{\lvert u \times v\rvert = \lvert u\rvert\lvert v\rvert \sin(t)}$

Use the right-hand rule for orientation. Reversing order negates the result.

Projection and perpendicular part

Component along v	$\mathbf{(u \cdot v)/\lvert v\rvert}$
Project u onto v	$\mathbf{w = [(u \cdot v)/(v \cdot v)]\, v}$
Perpendicular remainder	$\mathbf{u - w}$

The target vector v must be nonzero. The scalar component can be negative.

Lines and planes in space

Line through point P	$\mathbf{r(t) = P + t\, v}$
Plane through point P	$\mathbf{n \cdot (r - P) = 0}$
Distance to plane	$\mathbf{d = \lvert n \cdot (Q - P)\rvert/\lvert n\rvert}$

v is a nonzero direction vector. n is a nonzero normal. Q is the point measured.

Position, velocity and tangent

Velocity	$\mathbf{v(t) = r'(t)}$
Acceleration	$\mathbf{a(t) = r''(t)}$
Speed	$\mathbf{\lvert v(t)\rvert}$
Unit tangent, speed > 0	$\mathbf{T = v/\lvert v\rvert}$

Differentiate each position component. Integrate speed for total path length.

Direction, area and volume

Magnitude m and angle t	$\mathbf{(m \cos t,\ m \sin t)}$
Scale a vector	$\mathbf{\lvert cv\rvert = \lvert c\rvert\lvert v\rvert}$
Triangle with edges u, v	$\mathbf{\text{area} = \tfrac{1}{2}\lvert u \times v\rvert}$
Parallelepiped	$\mathbf{\text{volume} = \lvert u \cdot (v \times w)\rvert}$

Use components to choose the quadrant of a direction angle.