Vector Formulas

Components, magnitude and direction

From A to B **$\mathbf{v = B - A}$**

Length in 3D **$|\mathbf{v}| = \sqrt{(v_x^2 + v_y^2 + v_z^2)}$**

Unit direction, v != 0 **$\mathbf{u = v/|v|}$**

Vector sum **add matching components**

In 2D omit the z component. Multiplying by a negative scalar reverses direction.

Dot product and angle

Component product sum

$\mathbf{u \cdot v = u_x v_x + u_y v_y + u_z v_z}$

Angle for nonzero vectors

$\cos t = (u \cdot v)/(|u||v|)$

Perpendicular test **$\mathbf{u \cdot v = 0}$**

The perpendicular test describes an angle only when both vectors are nonzero.

Cross product in three dimensions

First component **$\mathbf{u_y v_z - u_z v_y}$**

Second component **$\mathbf{u_z v_x - u_x v_z}$**

Third component **$\mathbf{u_x v_y - u_y v_x}$**

Parallelogram area **$|u \times v| = |u||v|\sin(t)$**

Use the right-hand rule for orientation. Reversing order negates the result.

Projection and perpendicular part

Component along v **$\mathbf{(u \cdot v)/|v|}$**

Project u onto v **$\mathbf{w = [(u \cdot v)/(v \cdot v)]\, v}$**

Perpendicular remainder **$\mathbf{u - w}$**

The target vector v must be nonzero. The scalar component can be negative.

Lines and planes in space

Line through point P **$\mathbf{r(t) = P + t\, v}$**

Plane through point P **$\mathbf{n \cdot (r - P) = 0}$**

Distance to plane **$\mathbf{d = |n \cdot (Q - P)|/|n|}$**

v is a nonzero direction vector. n is a nonzero normal. Q is the point measured.

Position, velocity and tangent

Velocity **$\mathbf{v(t) = r'(t)}$**

Acceleration **$\mathbf{a(t) = r''(t)}$**

Speed **$\mathbf{|v(t)|}$**

Unit tangent, speed > 0 **$\mathbf{T = v/|v|}$**

Differentiate each position component. Integrate speed for total path length.

Direction, area and volume

Magnitude m and angle t **$\mathbf{(m \cos t, m \sin t)}$**

Scale a vector **$\mathbf{|cv| = |c||v|}$**

Triangle with edges u, v **$\mathbf{\text{area} = \tfrac{1}{2}\,|u \times v|}$**

Parallelepiped **$\mathbf{\text{volume} = |u \cdot (v \times w)|}$**

Use components to choose the quadrant of a direction angle.