Quadratic Formula Sheet

Find and count roots

Discriminant	$D = b^2 - 4ac$
Roots when D is nonnegative	$x = (-b \pm \sqrt{(D)})/(2a)$
Sum of roots	$r_1 + r_2 = -b/a$
Product of roots	$r_1\, r_2 = c/a$

For real coefficients, D > 0 gives two real roots, D = 0 one repeated root, D < 0 no real roots.

Quadratic forms

Standard	$y = ax^2 + bx + c$
Vertex	$y = a(x - h)^2 + k$
Factored if real roots exist	$y = a(x - r_1)(x - r_2)$
Axis in standard form	$x = -b/(2a)$

a is nonzero. The vertex is (h, k). Factored form shows real roots, including a repeated root.

Complete the square

Make the leading factor 1	$x^2 + 6x = 7$
Add the square of half of 6	$x^2 + 6x + 9 = 16$
Factor and solve	$(x + 3)^2 = 16;\ x = 1 \text{ or } -7$

If a is not 1, divide the equation by a first. Taking a square root can give two solutions.

Factor a trinomial

Leading coefficient 1 $x^2 + 5x + 6 = (x + 2)(x + 3)$

Split the middle term

$6x^2 + 11x + 3 = 6x^2 + 9x + 2x + 3$

Group and factor $6x^2 + 11x + 3 = (3x + 1)(2x + 3)$

For $ax^2 + bx + c$, seek two integers with product ac and sum b when possible.

Choose a quadratic method

Factors are visible	$x^2 - 7x + 12 = (x - 3)(x - 4)$
Already a square	$(x - 1)^2 = 9 \Rightarrow x = 4 \text{ or } -2$
Complex pair	$x^2 + 9 = 0 \Rightarrow x = \pm 3i$

Use the quadratic formula when factoring is not convenient. Set the equation equal to zero first.

Square patterns

Difference of squares	$a^2 - b^2 = (a - b)(a + b)$
Perfect square, plus	$a^2 + 2ab + b^2 = (a + b)^2$
Perfect square, minus	$a^2 - 2ab + b^2 = (a - b)^2$

Check the middle term before calling a trinomial a perfect square.