Probability Formulas

Basic probability

Equally likely outcomes	$P(A) = \text{favorable} / \text{total}$
Complement	$P(\text{not } A) = 1 - P(A)$
Example: fair die	$P(\text{even}) = 3/6 = ½$

The favorable count comes from the group named in the question.

Events and their complements

Complement	$P(\text{not } A) = 1 - P(A)$
A or B	$P(A) + P(B) - P(A \text{ and } B)$
Disjoint events	$P(A \text{ and } B) = 0$
Independent events	$P(A \text{ and } B) = P(A)P(B)$

Disjoint events with positive probabilities cannot be independent.

Conditional probability and Bayes

Given B	$P(A \mid B) = P(A \text{ and } B)/P(B)$
Reverse a conditional	$P(A \mid B) = P(B \mid A)P(A)/P(B)$
Split by a partition A_i	$P(B) = \sum P(B \mid A_i)P(A_i)$

Each conditioning event needs positive probability. Use all partition cases in the sum.

Counting and arrangements

Successive choices	multiply available counts
Order matters	$P(n,k) = n!/(n - k)!$
Order does not matter	$C(n,k) = n!/[k!(n - k)!]$

The formulas select k distinct objects from n, with 0 <= k <= n. Use 0! = 1.

Random variable mean and spread

Discrete mean	$E(X) = \sum x\, P(X = x)$
Variance	$\text{Var}(X) = E(X^2) - E(X)^2$
Linear transformation	$\text{Var}(aX + b) = a^2\, \text{Var}(X)$
Independent sum	$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

SD is the square root of variance. Expectation adds even without independence.

Binomial and geometric trials

Binomial: k successes	$P(X = k) = C(n,k)p^k(1 - p)^{n-k}$
Binomial moments	$E(X) = np$, $\text{Var}(X) = np(1 - p)$
First success at trial k	$P(X = k) = (1 - p)^{k-1}p$
Geometric mean	$E(X) = 1/p$

Use independent trials with constant success probability p. Geometric k starts at 1.

Poisson counts

Rate lambda per interval	$P(X = k) = e^{-\lambda}\lambda^k/k!$
Mean and variance	$E(X) = \text{Var}(X) = \lambda$
Example: no arrivals, rate 2	$P(X = 0) = e^{-2}$

Use k = 0, 1, 2, ... and lambda > 0. Match the rate to the interval length.

Continuous distributions

Normal standardization	$z = (x - \mu)/\sigma$
Uniform on [a,b]	$P(c \le X \le d) = (d - c)/(b - a)$
Exponential, x >= 0	$P(X > x) = e^{-\lambda x}$
Density gives interval area	$P(c \le X \le d) = \int_c^d f(x)\, dx$

For the uniform formula, [c,d] lies inside [a,b] and a < b.

For exponential waiting time lambda is positive. Density is not point probability.

Probability Formulas

Bernoulli and finite sampling

Bernoulli trial $P(X = 1) = p,\ P(X = 0) = 1 - p$

Hypergeometric count

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Continuous expectation $E(X) = \int x\, f(x)\, dx$

Exponential moments

$$E(X) = \frac{1}{\lambda},\ \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

Sample n from N with K successes, without replacement. The integral is over the support.