Precalculus Formula Sheet

Compose, invert and transform

Composition **$(f \text{ composed with } g)(x) = f(g(x))$**

Inverse on one-to-one domain **$g(f(x)) = x$**

Transform a parent function **$y = A\,f(B(x - h)) + k$**

In composition, g(x) must belong to the domain of f. For B != 0, move right by h and scale horizontal distances by 1/|B|.

Polynomials and rational graphs

Remainder on division by x − a **$r = P(a)$**

If P(a) = 0 **$x - a$ is a factor**

Rational graph **cancel common factors first**

Equal degrees **horizontal asymptote $y = a/b$**

a and b in the asymptote rule are leading coefficients. Keep original domain exclusions. Canceled denominator roots may be holes.

Exponential and logarithm laws

Inverse relationship **$\log_b(x) = y \text{ iff } b^y = x$**

Product and quotient

$\log_b(MN) = \log_b(M) + \log_b(N)$

Power rule **$\log_b(x^p) = p\log_b(x)$**

Change of base **$\log_b(x) = \ln(x)/\ln(b)$**

Use positive inputs and b > 0, b != 1. For quotients, subtract the logs.

Exponential models

General form **$y = a \times b^t$**

Growth **$b = 1 + r$**

Decay **$b = 1 - r$**

Example: 10% growth **$y = 40 \times 1.1^t$**

a is the initial value, t counts equal periods, and r is a per-period rate as a decimal. Use 0 < r < 1 for decay.

Quadratic equations

Standard form **$ax^2 + bx + c = 0$**

Discriminant **$D = b^2 - 4ac$**

Quadratic formula **$x = (-b \pm \text{sqrt}(D)) / (2a)$**

Axis of symmetry **$x = -b/(2a)$**

a is nonzero. For real coefficients, D < 0 means no real roots. sqrt(D) means square root.

Recognize centered conics

Circle **$(x - h)^2 + (y - k)^2 = r^2$**

Ellipse **$(x - h)^2/a^2 + (y - k)^2/b^2 = 1$**

Horizontal hyperbola **$(x - h)^2/a^2 - (y - k)^2/b^2 = 1$**

Vertical parabola **$(x - h)^2 = 4p(y - k)$**

The center is (h,k) for the first three. For the parabola it is the vertex.

Rational roots and binomial

Root p/q, reduced

p | constant, q | leading coefficient

Binomial expansion, n integer

$(a+b)^n = \text{sum } C(n,k)a^{n-k}b^k$

Compound balance **$A = P(1 + r/m)^{mt}$**

Use integer coefficients for the rational root test. r is annual, m counts periods per year, t is in years.

Sides, continuity and squeeze

Limits from both sides **$\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x)$**

Continuous at a **$\lim_{x \to a} f(x) = f(a)$**

Squeeze **$g(x) \le f(x) \le h(x)$**

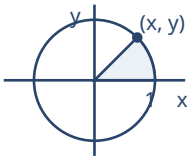
For a squeeze, both outer limits must agree. For continuity, f(a) must exist.

Precalculus Formula Sheet

Read the unit circle

Position at angle t	**(cos t, sin t)**
Circle equation	**$x^2 + y^2 = 1$**
Degrees to radians	**degrees × π/180**

Positive angles turn counterclockwise.



Pythagorean and reciprocal forms

Fundamental identity	**$\sin^2(t) + \cos^2(t) = 1$**
Divide by $\cos^2(t)$	**$1 + \tan^2(t) = \sec^2(t)$**
Divide by $\sin^2(t)$	**$1 + \cot^2(t) = \csc^2(t)$**
Quotient	**$\tan(t) = \sin(t)/\cos(t)$**

sec = 1/cos, csc = 1/sin and cot = cos/sin where defined.

Sum and difference of angles

Sine	**$\sin(a + b) = \sin a \cos b + \cos a \sin b$**
Cosine	**$\cos(a + b) = \cos a \cos b - \sin a \sin b$**

Tangent, p = tan a, q = tan b

$\tan(a + b) = (p + q)/(1 - pq)$

For a difference, replace b with −b. Use tangent only where all terms are defined.

Graph a sinusoid

Model	**$y = A \sin(B(x - h)) + k$**
Amplitude	**$\lvert A \rvert$**
Period, B != 0	**$2\pi/\lvert B \rvert$**
Phase shift and midline	**x shift h, y = k**

The period formula uses radians. A negative A reflects the wave across its midline.

Arithmetic sequences and sums

Constant difference d	**$a_n = a_1 + (n - 1)d$**
Sum of first n terms	**$S_n = n(a_1 + a_n)/2$**
Example: 4, 7, 10, …	**$a_8 = 25$, $S_8 = 116$**

Check the first differences before using the formula.

Geometric sequences and sums

Common ratio r	**$a_n = a_1 r^{n-1}$**
Finite sum, r != 1	**$S_n = a_1(1 - r^n)/(1 - r)$**
Finite sum, r = 1	**$S_n = n\,a_1$**
Infinite sum, $\lvert r \rvert < 1$	**$S = a_1/(1 - r)$**

The nth-term formula uses a_1 for n = 1, including when r = 0.

Double and half angles

Double sine	**$\sin(2t) = 2 \sin t \cos t$**
Double cosine	**$\cos(2t) = 1 - 2 \sin^2(t)$**
Half sine squared	**$\sin^2(t/2) = (1 - \cos t)/2$**
Half cosine squared	**$\cos^2(t/2) = (1 + \cos t)/2$**

For the unsquared half-angle values, select the sign from the quadrant of t/2.

Non-right triangle laws

Sine law	**$a/\sin A = b/\sin B = c/\sin C$**
Cosine law	**$c^2 = a^2 + b^2 - 2ab \cos C$**
Area from two sides	**$K = ½\ ab \sin C$**

Each capital angle is opposite its lowercase side. With two sides and a nonincluded angle, check for zero, one or two triangles.