Matrix Formulas

Matrix operations and shapes

Addition **add entries for equal shapes**

Scalar multiplication

multiply each entry by the scalar

Transpose **rows become columns**

Product transpose $(AB)^T = B^T A^T$

The transpose of an m by n matrix is n by m.

Multiply row by column

Compatible shapes **(m by n)(n by p) = m by p**

Entry at row i, column j

dot row i of A with column j of B

Example row and column **[2,3] dot [4,5] = 23**

Identity matrix **AI = A, IA = A**

Choose the identity with the matching dimension. AB and BA can differ or be undefined.

2 by 2 determinants and inverses

Rows [a,b] and [c,d] **D = ad − bc**

First row of inverse **[d, −b]/D**

Second row of inverse **[−c, a]/D**

A square matrix has an inverse exactly when its determinant is nonzero.

3 by 3 determinant and scaling

Cofactors across row 1 $\det = a M_1 - b M_2 + c M_3$

Row exchange **flips determinant sign**

Multiply one row by k **multiplies determinant by k**

Product, square matrices **det(AB) = det(A) det(B)**

a, b and c are first-row entries. M_j is the 2 by 2 minor determinant at column j.

Adding a multiple of one row to another leaves the determinant unchanged.

Inverse and linear systems

Augment and row reduce

[A | I] becomes [I | inverse A]

Solve Ax = b if A invertible **x = (inverse A)b**

Cramer, square invertible A $x_i = \det(A_i)/\det(A)$

General case **row reduce [A | b]**

A_i replaces column i of A with b. Read inconsistent and free-variable rows before solving.

Eigenvalues and eigenvectors

Eigenvalue equation $Av = \lambda v, v \neq 0$

Find eigenvalues $\det(A - \lambda I) = 0$

Find eigenvectors $(A - \lambda I)v = 0$

Triangular matrix

eigenvalues are diagonal entries

For each eigenvalue, keep the nonzero vectors in its null space.

Determinant and inverse identities

Transpose $\det(A^T) = \det(A)$

Scale square n by n matrix $\det(cA) = c^n \det(A)$

Inverse from cofactors **inverse(A) = adj(A)/det(A)**

Product inverse

inverse(AB) = inverse(B) inverse(A)

adj(A) is the transpose of the cofactor matrix. Require invertibility where used.

Trace and determinant checks

Trace **tr(A) = sum diagonal entries**

Sum of eigenvalues $\text{tr}(A) = \text{sum lambda}_i$

Product of eigenvalues $\det(A) = \text{product lambda}_i$

Count complex roots too, with their algebraic multiplicities.