Logarithm Rules Cheat Sheet

Logs undo exponentiation

Definition	$\log_b(x) = y \text{ iff } b^y = x$
Identity and zero	$\log_b(b) = 1;\ \log_b(1) = 0$
Change of base	$\log_b(x) = \ln(x)/\ln(b)$
Natural log	$\ln(e^x) = x$

Real logs have positive inputs and a positive base other than 1. The natural log has base e.

Core logarithm laws

Product	$\log_b(MN) = \log_b(M) + \log_b(N)$
Quotient	$\log_b(M/N) = \log_b(M) - \log_b(N)$
Power	$\log_b(M^p) = p\,\log_b(M)$

Use a positive base other than 1 and positive M and N. There is no sum law.

Solve and check log equations

Exponentiate	$\log_2(x - 1) = 3 \Rightarrow x - 1 = 8$
Check the input	$x = 9;\ x - 1 = 8 > 0$
Derivative connection	$d[\ln(x)]/dx = 1/x \text{ for } x > 0$

Before combining logs, write down every original domain restriction.

Solve an exponential equation

Rewrite using one base	$9^x = 27 \Rightarrow 3^{2x} = 3^3$
Match exponents	$2x = 3;\ x = 3/2$
If bases do not match	$2^x = 5 \Rightarrow x = \ln(5)/\ln(2)$

Use a positive base other than 1 and a positive right-hand side.