Limits Cheat Sheet

Limit laws

Sum and difference	$\lim(f \pm g) = L \pm M$
Constant multiple	$\lim(cf) = cL$
Product	$\lim(fg) = LM$
Quotient, M != 0	$\lim(f/g) = L/M$

These rules use finite limits L = lim f and M = lim g at the same point.

Sides, continuity and squeeze

Limits from both sides	$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
Continuous at a	$\lim_{x \to a} f(x) = f(a)$
Squeeze	$g(x) \le f(x) \le h(x)$

For a squeeze, both outer limits must agree. For continuity, f(a) must exist.

Useful small-input limits

As x approaches 0	$\sin(x)/x \to 1$
As x approaches 0	$(\cos(x) - 1)/x \to 0$
As x approaches 0	$(e^x - 1)/x \to 1$
As x approaches 0	$\ln(1 + x)/x \to 1$

Use radians in the trigonometric limits.

When substitution gives 0/0

Factor first	$(x^2 - 9)/(x - 3) = x + 3$
At x = 3	limit = 6
Rationalize a root	$\sqrt{(1 + x)} - 1 = x/(\sqrt{(1 + x)} + 1)$

The cancellation uses x != 3. A limit can exist at a hole.

Indeterminate quotients

Eligible forms	$0/0$ or ∞/∞
Compare limits	$\lim(f/g) = \lim(f'/g')$
Example near 0	$\lim (e^x - 1)/x = \lim e^x = 1$

Require f and g differentiable nearby, g′ != 0, and a limit of f′/g′.
Rewrite 0 times infinity, infinity minus infinity or a power before applying.

End behavior

Lower degree on top	$\lim P(x)/Q(x) = 0$
Equal degrees	$\lim P(x)/Q(x) = a/b$
Higher degree on top	**divide to find the dominant term**

Here x tends to either infinity, and a and b are leading coefficients.
An infinite limit describes unbounded behavior. It is not a real value.

Derivative from a limit

Definition at x	$f'(x) = \lim [f(x + h) - f(x)]/h$
For $f(x) = x^2$	$[f(x + h) - f(x)]/h = 2x + h$
As h approaches 0	$f'(x) = 2x$

Take the limit as h approaches 0 through nonzero h.