# Limits Cheat Sheet

## Limit laws

| | |
|---|---|
| Sum and difference | $\lim(f \pm g) = L \pm M$ |
| Constant multiple | $\lim(cf) = cL$ |
| Product | $\lim(fg) = LM$ |
| Quotient, M != 0 | $\lim(f/g) = L/M$ |

These rules use finite limits L = lim f and M = lim g at the same point.

## Sides, continuity and squeeze

| | |
|---|---|
| Limits from both sides | $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ |
| Continuous at a | $\lim_{x \to a} f(x) = f(a)$ |
| Squeeze | $g(x) \le f(x) \le h(x)$ |

For a squeeze, both outer limits must agree. For continuity, f(a) must exist.

## Useful small-input limits

| | |
|---|---|
| As x approaches 0 | $\sin(x)/x \to 1$ |
| As x approaches 0 | $(\cos(x) - 1)/x \to 0$ |
| As x approaches 0 | $(e^x - 1)/x \to 1$ |
| As x approaches 0 | $\ln(1 + x)/x \to 1$ |

Use radians in the trigonometric limits.

## When substitution gives 0/0

| | |
|---|---|
| Factor first | $(x^2 - 9)/(x - 3) = x + 3$ |
| At x = 3 | limit = 6 |
| Rationalize a root | $\sqrt{(1 + x)} - 1 = x/(\sqrt{(1 + x)} + 1)$ |

The cancellation uses x != 3. A limit can exist at a hole.

## Indeterminate quotients

| | |
|---|---|
| Eligible forms | 0/0 or ∞/∞ |
| Compare limits | $\lim(f/g) = \lim(f'/g')$ |
| Example near 0 | $\lim (e^x - 1)/x = \lim e^x = 1$ |

Require f and g differentiable nearby, g′ != 0, and a limit of f′/g′.

Rewrite 0 times infinity, infinity minus infinity or a power before applying.

## End behavior

| | |
|---|---|
| Lower degree on top | $\lim P(x)/Q(x) = 0$ |
| Equal degrees | $\lim P(x)/Q(x) = a/b$ |
| Higher degree on top | **divide to find the dominant term** |

Here x tends to either infinity, and a and b are leading coefficients.

An infinite limit describes unbounded behavior. It is not a real value.

## Derivative from a limit

| | |
|---|---|
| Definition at x | $f'(x) = \lim [f(x + h) - f(x)]/h$ |
| For $f(x) = x^2$ | $[f(x + h) - f(x)]/h = 2x + h$ |
| As h approaches 0 | $f'(x) = 2x$ |

Take the limit as h approaches 0 through nonzero h.