Integral Table and Integration Formulas

Power and reciprocal integrals

Power, n != −1	$\int x^n\,dx = x^{n+1}/(n+1) + C$
Reciprocal, x != 0	$\int dx/x = \ln\lvert x\rvert + C$
Constant	$\int k\,dx = kx + C$

Use the power rule on intervals where both sides are defined.

Exponentials and logarithms

Natural exponential	$\int e^x\,dx = e^x + C$
General base	$\int a^x\,dx = a^x/\ln(a) + C$
Natural log, x > 0	$\int \ln(x)\,dx = x\ln(x) - x + C$

The exponential base satisfies a > 0 and a != 1.

Trigonometric integrals

Sine	$\int \sin(x)\,dx = -\cos(x) + C$
Cosine	$\int \cos(x)\,dx = \sin(x) + C$
Secant squared	$\int \sec^2(x)\,dx = \tan(x) + C$
Cosecant squared	$\int \csc^2(x)\,dx = -\cot(x) + C$

Use radians. Work within an interval that avoids poles.

More trigonometric integrals

Secant times tangent

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

Cosecant times cotangent

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

Tangent	$\int \tan(x)\,dx = -\ln\lvert\cos(x)\rvert + C$
Secant	$\int \sec(x)\,dx = \ln\lvert\sec(x)+\tan(x)\rvert + C$

Check the answer by differentiation on the same interval.

Inverse-trig patterns

Positive quadratic	$\int dx/(a^2+x^2) = \operatorname{atan}(x/a)/a + C$
Positive radical	$\int dx/\sqrt{a^2-x^2} = \operatorname{asin}(x/a) + C$
Arcsec pattern	$\int dx/(x\sqrt{x^2-a^2})$
Arcsec result	$\operatorname{arcsec}(\lvert x\rvert/a)/a + C$

Take a > 0. Use |x| < a for arcsine and |x| > a for the arcsec pattern.

Choose an integration method

Inside and its derivative	$u = g(x),\ du = g'(x)\,dx$
Product, choose u and dv	$\int u\,dv = uv - \int v\,du$
Rational function	divide first if improper
Decompose factors	$A/(x-a) + B/(x-b)$

The decomposition shown uses two distinct linear factors.

For definite substitution, update both bounds or return to x.

Trigonometric substitutions

sqrt(a² − x²)	$x = a\sin(t),\ \lvert t\rvert < \pi/2$
sqrt(a² + x²)	$x = a\tan(t),\ \lvert t\rvert < \pi/2$
sqrt(x² − a²), x > a	$x = a\sec(t),\ 0 < t < \pi/2$

Take a > 0. Use a separate branch for x < −a. Substitute dx as well.

Accumulation and change

Integral of a derivative	$\int_a^b F'(x)\,dx = F(b) - F(a)$
Accumulation derivative	$d/dx \int_a^x f(t)\,dt = f(x)$
Mean value of f	$f_{avg} = \int_a^b f(x)\,dx / (b-a)$

Use continuous f for the accumulation and average formulas, with a < b.

Integral Table and Integration Formulas

Linearity and substitution

Constant multiple $\int k\,f(x)\,dx = k\int f(x)\,dx$

Sum or difference $\int (f \pm g)\,dx = \int f\,dx \pm \int g\,dx$

Exponential inside $\int e^{u}\,u'\,dx = e^{u} + C$

Logarithmic pattern $\int u'/u\,dx = \ln|u| + C$

For the last pattern, u must stay nonzero on the interval.

Approximation and improper limits

Uniform width $dx = (b - a)/n$

Trapezoids $T = dx[f_0/2 + \text{sum inner } f + f_n/2]$

Unbounded interval $\int_a^{\infty} f = \lim \int_a^b f$

The last limit is as b tends to infinity. Split at interior singularities.

Special definite integral results

Mean value for continuous f

$\int_a^b f(x)\,dx = f(c)(b - a)$

Gaussian over the real line $\int_{-\infty}^{\infty} e^{-x^2}\,dx$

Gaussian result $\sqrt{(\pi)}$

Logarithm from cotangent

$\int \cot(x)\,dx = \ln|\sin(x)| + C$

For the mean value theorem, a < b and some c lies in [a,b].

Cosecant and applications

Cosecant $\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$

Mean value $f_{avg} = \int_a^b f(x)\,dx/(b - a)$

Reverse the bounds $\int_b^a f\,dx = -\int_a^b f\,dx$

Use each formula on intervals where the relevant functions are defined.