Inequality Rules Cheat Sheet

Move terms without losing order

Add the same term	$a < b \Rightarrow a + c < b + c$
Multiply by $c > 0$	$a < b \Rightarrow ac < bc$
Multiply by $c < 0$	$a < b \Rightarrow ac > bc$
Transitive comparison	$a < b < c \Rightarrow a < c$

Dividing by a negative also reverses order. Multiplying by zero erases the comparison.

Flip the sign when needed

Divide both sides by −3	$-3x \le 12 \Rightarrow x \ge -4$
Check the endpoint	$-3(-4) = 12$
Check one allowed value	$x = 0$ gives $0 \le 12$

Reverse the comparison only when multiplying or dividing both sides by a negative number.

Read compound & absolute value

Both conditions	$-2 \le x < 4$ means $[-2, 4)$
Inside, $r > 0$	$\lvert x - h \rvert < r \Rightarrow h - r < x < h + r$
Outside, $r > 0$	$\lvert x - h \rvert > r \Rightarrow x < h - r$ or $x > h + r$

Use a closed endpoint when equality is allowed. “And” means overlap. “Or” means union.

Use a sign chart

Roots split the line	$(x - 2)(x + 1) > 0$
Test the outer intervals	$x < -1$ or $x > 2$
Rational expression	$(x - 1)/(x + 2) \ge 0$
Allow 1, exclude −2	$x < -2$ or $x \ge 1$

Check zeros of the numerator and denominator separately. Never include a zero denominator.

Useful bounds

Triangle inequality	$\lvert a + b \rvert \le \lvert a \rvert + \lvert b \rvert$
Arithmetic and geometric means	$(a + b)/2 \ge \sqrt{(ab)}$
Equality in the mean bound	$a = b$

The arithmetic-geometric mean bound here uses nonnegative a and b.