Geometry Formula Sheet: Expanded

Perimeter of straight sides

Triangle	$P = a + b + c$
Rectangle or parallelogram	$P = 2(a + b)$
Trapezoid	$P = a + b + c + d$
Regular n-gon	$P = ns$

Use side lengths, not perpendicular height. Trace each boundary edge exactly once.

Choose a triangle area rule

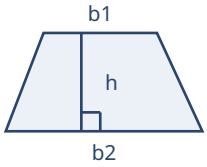
Base and perpendicular height	$A = bh/2$
Two sides, included angle	$A = ab \sin(C)/2$
Three sides	$s = (a + b + c)/2$
Heron	$A = \sqrt{(s(s - a)(s - b)(s - c))}$

The three side lengths must form a triangle. C is the angle between sides a and b.

Area of plane figures

Triangle	$A = ½bh$
Rectangle or parallelogram	$A = bh$
Trapezoid	$A = ½(b_1 + b_2)h$

Measure h perpendicular to the base.
The trapezoid bases are parallel.
Use square units.



Polygons and diagonals

Rhombus or kite	$A = d_1\, d_2/2$
Regular polygon	$A = ap/2$

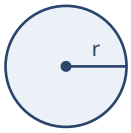
Example: apothem 6, perimeter 40
$A = 120$ square units

The regular polygon apothem a is perpendicular to a side. Use the full perimeter p.

Circles, rings and sectors

Disk	$A = \pi r^2$
Annulus	$A = \pi(R^2 - r^2)$
Sector in radians	$A = r^2\theta/2$
Sector in degrees	$A = (\theta/360°)\pi r^2$

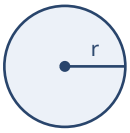
R is the outer radius. Use the formula that matches the angle units.



Circles, arcs & sector borders

Full circle	$C = 2\pi r = \pi d$
Arc in radians	$s = r\theta$
Arc in degrees	$s = (\theta/360°)2\pi r$
Sector boundary	$P = s + 2r$

A sector smaller than a full turn includes two radii. Arc length uses linear units.



Coordinate geometry

Slope	$m = (y_2 - y_1) / (x_2 - x_1)$
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$
Squared distance	$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Take the nonnegative square root to get d. A vertical line has no defined slope.

Area from coordinates

List ordered polygon vertices	$(x_1,y_1), \ldots, (x_n,y_n)$
Shoelace	$A = ½ \lvert\sum(x_i\, y_{i+1} - y_i\, x_{i+1})\rvert$
Example: (0,0), (4,0), (0,3)	$A = ½ \times 12 = 6$

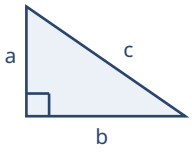
Walk around a simple polygon in order and return to the first vertex for the last product.

Geometry Formula Sheet: Expanded

Right triangles

Pythagorean theorem $a^2 + b^2 = c^2$

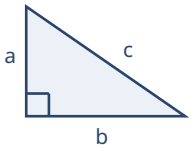
Legs a and b meet at a right angle.
Hypotenuse c is opposite that angle.



Special triangles and angle rules

45°–45°–90° side ratios	$1 : 1 : \sqrt{2}$
30°–60°–90° side ratios	$1 : \sqrt{3} : 2$
Triangle interior sum	$A + B + C = 180°$
Polygon interior sum	$(n - 2)180°$

In a 30°–60°–90° triangle, the shortest side is opposite 30°.



Scale and similarity

Length	$\text{new} = k \times \text{original}$
Area	$\text{new} = k^2 \times \text{original}$
Volume	$\text{new} = k^3 \times \text{original}$

k is a positive scale factor. Match corresponding sides before finding the ratio.

Surface area

Right prism, lateral	$S = Ph$
Right prism, total	$S = Ph + 2B$
Right cylinder, lateral	$S = 2\pi rh$
Right cylinder, total	$S = 2\pi rh + 2\pi r^2$

P is base perimeter; B is base area. Total adds both bases. Use square units.



Pyramids & right cones

Regular pyramid, lateral	$S = P\ell/2$
Right cone, lateral	$S = \pi r\ell$
Right cone, total	$S = \pi r\ell + \pi r^2$
Right cone slant height	$\ell = \sqrt{r^2 + h^2}$

Add base area for total pyramid surface area. Use slant height for the lateral face and perpendicular height for the solid.

Volume

Prism or cylinder	$V = Bh$
Pyramid or cone	$V = Bh / 3$
Sphere	$V = (4/3)\pi r^3$

B is the area of one base. h is perpendicular height. Use cubic units.

Choose base and height

Cylinder: r = 3 cm, h = 5 cm	$V = \pi(3)^2(5) = 45\pi \text{ cm}^3$
Cone with the same base and height	$V = 15\pi \text{ cm}^3$
Hemisphere	$V = 2\pi r^3/3$

Measure the perpendicular height. If a diameter is given, halve it before squaring the radius.



Frustum, ellipsoid & torus

Cone frustum	$V = \pi h(R^2 + Rr + r^2)/3$
Ellipsoid, positive semiaxes	$V = 4\pi abc/3$
Ring torus, R > r	$V = 2\pi^2 Rr^2$

In the frustum, h is perpendicular height. A torus uses centerline radius R and tube radius r.