Distance and Midpoint Formula Sheet

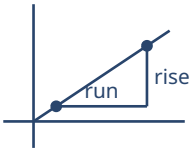
Distance in 2D or 3D

Planar distance $d = \sqrt{(\Delta x^2 + \Delta y^2)}$

Space distance $d = \sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)}$

Example: (1,2) to (4,6) $d = \sqrt{(3^2 + 4^2)} = 5$

Take the second coordinate minus the first for each change. The square root is nonnegative.



Midpoint and missing endpoint

Halfway coordinate by coordinate $M = (A + B)/2$

Recover the second endpoint $B = 2M - A$

Example: A = (2,1), M = (5,3) $B = (8,5)$

Treat the points as coordinate vectors. The same rule works in 3D.

Partition a segment

Point along A to B $P = A + t(B - A)$

Internal ratio AP:PB = m:n $t = m/(m + n)$

Example: 2:1 from (0,0) to (9,6) $P = (6,4)$

A point strictly inside has $0 < t < 1$. Use t outside that interval for external division.

Distance to a line or plane

Line Ax + By + C = 0 $d = |Ax_0 + By_0 + C|/\sqrt{(A^2 + B^2)}$

Plane Ax + By + Cz + D = 0

$d = |Ax_0 + By_0 + Cz_0 + D|$

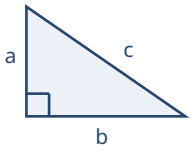
Divide the plane numerator by $\sqrt{(A^2 + B^2 + C^2)}$

The coefficient vector must be nonzero. These are shortest, perpendicular distances.

Right triangles

Pythagorean theorem $a^2 + b^2 = c^2$

Legs a and b meet at a right angle. Hypotenuse c is opposite that angle.



Example: Find the slope

Points (1, 2) and (5, 4) $m = (4 - 2) / (5 - 1) = ½$

Read the vertical change first, then the horizontal change. Keep the point order the same.

