Calculus Formula Sheet

Sides, continuity and squeeze

Limits from both sides	$\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x)$
Continuous at a	$\lim_{x \to a} f(x) = f(a)$
Squeeze	$g(x) \le f(x) \le h(x)$

For a squeeze, both outer limits must agree. For continuity, f(a) must exist.

Derivative from a limit

Definition at x	$f'(x) = \lim [f(x + h) - f(x)]/h$
For f(x) = x^2	$[f(x + h) - f(x)]/h = 2x + h$
As h approaches 0	$f'(x) = 2x$

Take the limit as h approaches 0 through nonzero h.

Indeterminate quotients

Eligible forms	$0/0$ or ∞/∞
Compare limits	$\lim(f/g) = \lim(f'/g')$
Example near 0	$\lim (e^x - 1)/x = \lim e^x = 1$

Require f and g differentiable nearby, g′ != 0, and a limit of f′/g′.
Rewrite 0 times infinity, infinity minus infinity or a power before applying.

Derivative rules at a glance

Power, n positive integer	$(x^n)' = n\,x^{n-1}$
Product	$(fg)' = f'g + fg'$
Quotient, g != 0	$(f/g)' = (f'g - fg')/g^2$
Chain	$(f(g(x)))' = f'(g(x))g'(x)$

Use the chain rule when one function is inside another.

Common trig derivatives

Sine	$(\sin x)' = \cos x$
Cosine	$(\cos x)' = -\sin x$
Tangent	$(\tan x)' = \sec^2 x$

Angles are in radians. Tangent is differentiable where cos x is nonzero.

Exponential and log derivatives

Natural exponential	$(\exp(x))' = \exp(x)$
Exponential base a	$(a^x)' = a^x \ln(a)$
Natural logarithm	$(\ln x)' = 1/x$

exp(x) means e to the power x. Use a > 0 for a^x and x > 0 for ln x.

Power and reciprocal integrals

Power, n != −1	$\int x^n\,dx = x^{n+1}/(n + 1) + C$
Reciprocal, x != 0	$\int dx/x = \ln\lvert x\rvert + C$
Constant	$\int k\,dx = kx + C$

Use the power rule on intervals where both sides are defined.

Accumulation and change

Integral of a derivative	$\int_a^b F'(x)\,dx = F(b) - F(a)$
Accumulation derivative	$d/dx \int_a^x f(t)\,dt = f(x)$
Mean value of f	$f_{avg} = \int_a^b f(x)\,dx / (b - a)$

Use continuous f for the accumulation and average formulas, with a < b.

Calculus Formula Sheet

Choose an integration method

Inside and its derivative	$u = g(x),\ du = g'(x)\,dx$
Product, choose u and dv	$\int u\,dv = uv - \int v\,du$
Rational function	**divide first if improper**
Decompose factors	$A/(x - a) + B/(x - b)$

The decomposition shown uses two distinct linear factors.
For definite substitution, update both bounds or return to x.

Trigonometric integrals

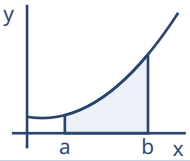
Sine	$\int \sin(x)\,dx = -\cos(x) + C$
Cosine	$\int \cos(x)\,dx = \sin(x) + C$
Secant squared	$\int \sec^2(x)\,dx = \tan(x) + C$
Cosecant squared	$\int \csc^2(x)\,dx = -\cot(x) + C$

Use radians. Work within an interval that avoids poles.

Area, volume and distance

Area between curves	$A = \int_a^b \lvert f(x) - g(x)\rvert\,dx$
Washer cross-section	$V = \pi \int_a^b (R^2 - r^2)\,dx$
Total distance	$D = \int_a^b \lvert v(t)\rvert\,dt$
Arc length (AP Calculus BC)	$L = \int_a^b \sqrt{(1 + f'(x)^2)}\,dx$

R and r are the outer and inner radii.



Useful small-input limits

As x approaches 0	$\sin(x)/x \to 1$
As x approaches 0	$(\cos(x) - 1)/x \to 0$
As x approaches 0	$(e^x - 1)/x \to 1$
As x approaches 0	$\ln(1 + x)/x \to 1$

Use radians in the trigonometric limits.

Differential equation models

Separable equation	$y' = g(x)\,h(y)$
Where h(y) != 0	$\int dy/h(y) = \int g(x)\,dx + C$
Exponential growth or decay	$y' = ky,\ y = y_0\,e^{kt}$
Euler step	$y_{next} = y + h\,f(x,y)$

When separating, also check constant solutions at zeros of h(y).

Geometric sequences and sums

Common ratio r	$a_n = a_1\,r^{n-1}$
Finite sum, r != 1	$S_n = a_1(1 - r^n)/(1 - r)$
Finite sum, r = 1	$S_n = n\,a_1$
Infinite sum, |r| < 1	$S = a_1/(1 - r)$

The nth-term formula uses a_1 for n = 1, including when r = 0.

Convergence tests

Term limit nonzero or absent	**series diverges**
p-series, sum 1/n^p	**converges exactly when p > 1**
Ratio L = lim |a_next/a_n|	
L < 1 converges absolutely	
Ratio L > 1	**diverges. L = 1 is inconclusive.**

A zero term limit alone does not show convergence.

Taylor series and radius

Taylor coefficient at a	$c_k = D^k\,f(a)/k!$
Power series	$\sum c_k\,(x - a)^k$
Inside radius	$\lvert x - a\rvert < R$
Remainder bound	$\lvert R_n\rvert \le M\lvert x - a\rvert^{n+1}/(n + 1)!$

D^k means the kth derivative. Check that the Taylor remainder tends to zero.
Test endpoints separately. M bounds the size of derivative n + 1 between a and x.