AP Calculus AB and BC Formula Sheet

Sides, continuity and squeeze

Limits from both sides	$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
Continuous at a	$\lim_{x \to a} f(x) = f(a)$
Squeeze	$g(x) \le f(x) \le h(x)$

For a squeeze, both outer limits must agree. For continuity, f(a) must exist.

Indeterminate quotients

Eligible forms	$0/0$ or ∞/∞
Compare limits	$\lim(f/g) = \lim(f'/g')$
Example near 0	$\lim (e^x - 1)/x = \lim e^x = 1$

Require f and g differentiable nearby, $g' \neq 0$, and a limit of f'/g'.
Rewrite 0 times infinity, infinity minus infinity or a power before applying.

Derivative rules at a glance

Power, n positive integer	$(x^n)' = n\,x^{n-1}$
Product	$(fg)' = f'g + fg'$
Quotient, $g \neq 0$	$(f/g)' = (f'g - fg')/g^2$
Chain	$(f(g(x)))' = f'(g(x))g'(x)$

Use the chain rule when one function is inside another.

AB: definitions and methods

Derivative at x	$\lim_{h \to 0} [f(x + h) - f(x)]/h$
Substitution	$u = g(x);\ du = g'(x)\,dx$
Example: $u = x^2$	$\int 2x \cos(x^2)\,dx = \sin(x^2) + C$

Intermediate value theorem
continuous f reaches intermediate values

For a definite substitution, change both bounds to u or return to x.

Tangent and extrema checks

Linearization at a	$L(x) = f(a) + f'(a)(x - a)$
Mean value theorem	$f'(c) = [f(b) - f(a)]/(b - a)$
Critical point, interior	$f'(c) = 0$ or $f'(c)$ undefined

For MVT, f is continuous on [a,b] and differentiable on (a,b).
If $f'(c) = 0$, $f''(c) > 0$ gives a local min and $f''(c) < 0$ a local max.

Antiderivatives for exam practice

Power, $n \neq -1$	$\int x^n\,dx = x^{n+1}/(n+1) + C$
Reciprocal	$\int dx/x = \ln\lvert x\rvert + C$
Natural exponential	$\int e^x\,dx = e^x + C$
Sine	$\int \sin x\,dx = -\cos x + C$
Cosine	$\int \cos x\,dx = \sin x + C$

Work on intervals where the integrands are defined.

Accumulation and change

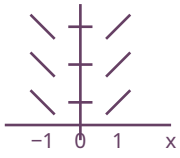
Integral of a derivative	$\int_a^b F'(x)\,dx = F(b) - F(a)$
Accumulation derivative	$d/dx \int_a^x f(t)\,dt = f(x)$
Mean value of f	$f_{avg} = \int_a^b f(x)\,dx / (b - a)$

Use continuous f for the accumulation and average formulas, with a < b.

AB: differential equations

At a slope-field point	$\text{slope} = dy/dx$
Separate $y' = g(x)\,h(y)$	$\int dy/h(y) = \int g(x)\,dx + C$
Example: $y' = x$, $y(0) = 1$	$y = x^2/2 + 1$

The initial condition fixes C. At zeros of h, also check constant solutions.



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Trigonometric derivatives

Sine	$(\sin x)' = \cos x$
Cosine	$(\cos x)' = -\sin x$
Tangent	$(\tan x)' = \sec^2 x$
Secant	$(\sec x)' = \sec x \tan x$
Cosecant	$(\csc x)' = -\csc x \cot x$
Cotangent	$(\cot x)' = -\csc^2 x$

Use radians and keep the inputs in the domain.

Implicit and inverse derivatives

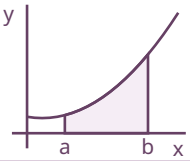
Implicit example	$x^2 + y^2 = 25$
Differentiate both sides	$2x + 2yy' = 0$
If y != 0	$y' = -x/y$
Inverse g of f	$g'(x) = 1/f'(g(x))$

For an inverse derivative, work on an invertible branch with nonzero f′.

Area, volume and distance

Area between curves	$A = \int_a^b \lvert f(x) - g(x) \rvert \, dx$
Washer cross-section	$V = \pi \int_a^b (R^2 - r^2) \, dx$
Total distance	$D = \int_a^b \lvert v(t) \rvert \, dt$
Arc length (AP Calculus BC)	$L = \int_a^b \sqrt{1 + f'(x)^2} \, dx$

R and r are the outer and inner radii.



BC: beyond basic substitution

Integration by parts	$\int u \, dv = uv - \int v \, du$

Unbounded at an interior point

split and take one-sided limits

Logistic model, capacity K	$y' = ky(1 - y/K)$
Euler step with step size h	$y_{next} = y + h\, f(x,y)$
Polar area, radians	$A = \frac{1}{2} \int_a^b r(t)^2 \, dt$

Every improper part must converge. Sweep the polar region once.

BC: parametric and polar

Slope, dx/dt != 0	$dy/dx = (dy/dt)/(dx/dt)$
Speed	$\sqrt{(dx/dt)^2 + (dy/dt)^2}$
Polar slope numerator	$r' \sin t + r \cos t$
Polar slope denominator	$r' \cos t - r \sin t$

Polar slope divides those two expressions if the denominator is nonzero.

For path length, integrate speed over time.

BC: choose a series test

Term limit nonzero or absent	**series diverges**
p-series	$\sum 1/n^p$ **converges for p > 1**

Ratio or root test result L

L < 1 converges absolutely

Alternating series error

|error| ≤ first omitted magnitude

L > 1 diverges and L = 1 is inconclusive. For alternating bounds, magnitudes decrease to 0.

Taylor series and radius

Taylor coefficient at a	$c_k = D^k f(a)/k!$
Power series	$\sum c_k (x - a)^k$
Inside radius	$\lvert x - a \rvert < R$
Remainder bound	$\lvert R_n \rvert \le M \lvert x - a \rvert^{n+1}/(n+1)!$

D^k means the kth derivative. Check that the Taylor remainder tends to zero.

Test endpoints separately. M bounds the size of derivative n + 1 between a and x.

Maclaurin series to recognize

Exponential	$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$
Sine	$\sin x = x - x^3/3! + x^5/5! - \ldots$
Cosine	$\cos x = 1 - x^2/2! + x^4/4! - \ldots$
Geometric	$1/(1 - x) = 1 + x + x^2 + \ldots$

The first three converge for all real x. The last requires $\lvert x \rvert < 1$.