Algebra Formula Sheet

Three forms of a line

Slope-intercept	$y = mx + b$
Point-slope	$y - y_1 = m(x - x_1)$
Standard	$Ax + By = C$

Read m and b in slope-intercept form. Use point-slope when a point and slope are given.
A and B are not both zero. A vertical line cannot be written as y = mx + b.

Coordinate geometry

Slope	$m = (y_2 - y_1) / (x_2 - x_1)$
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$
Squared distance	$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Take the nonnegative square root to get d. A vertical line has no defined slope.

Find and count roots

Discriminant	$D = b^2 - 4ac$
Roots when D is nonnegative	$x = (-b \pm \sqrt{(D)})/(2a)$
Sum of roots	$r_1 + r_2 = -b/a$
Product of roots	$r_1 r_2 = c/a$

For real coefficients, D > 0 gives two real roots, D = 0 one repeated root, D < 0 no real roots.

Multiply or divide powers

Same base, multiply	$a^m a^n = a^{m+n}$
Same base, divide	$a^m/a^n = a^{m-n}$
Power of a power	$(a^m)^n = a^{mn}$
Power of a product	$(ab)^n = a^n b^n$

For these integer-exponent rules, assume nonzero bases. Add exponents only in a same-base product.

Square patterns

Difference of squares	$a^2 - b^2 = (a - b)(a + b)$
Perfect square, plus	$a^2 + 2ab + b^2 = (a + b)^2$
Perfect square, minus	$a^2 - 2ab + b^2 = (a - b)^2$

Check the middle term before calling a trinomial a perfect square.

Core logarithm laws

Product	$\log_b(MN) = \log_b(M) + \log_b(N)$
Quotient	$\log_b(M/N) = \log_b(M) - \log_b(N)$
Power	$\log_b(M^p) = p \log_b(M)$

Use a positive base other than 1 and positive M and N. There is no sum law.

Arithmetic sequences and sums

Constant difference d	$a_n = a_1 + (n - 1)d$
Sum of first n terms	$S_n = n(a_1 + a_n)/2$
Example: 4, 7, 10, ...	$a_8 = 25, S_8 = 116$

Check the first differences before using the formula.

Geometric sequences and sums

Common ratio r	$a_n = a_1 r^{n-1}$
Finite sum, r != 1	$S_n = a_1(1 - r^n)/(1 - r)$
Finite sum, r = 1	$S_n = n a_1$
Infinite sum, |r| < 1	$S = a_1/(1 - r)$

The nth-term formula uses a_1 for n = 1, including when r = 0.