# ACT Math Formula Sheet

## Three forms of a line

| | |
|---|---|
| Slope-intercept | $y = mx + b$ |
| Point-slope | $y - y_1 = m(x - x_1)$ |
| Standard | $Ax + By = C$ |

Read m and b in slope-intercept form. Use point-slope when a point and slope are given.
A and B are not both zero. A vertical line cannot be written as y = mx + b.

## Linear equations and systems

| | |
|---|---|
| Slope-intercept | $y = mx + b$ |
| Point-slope | $y - y_1 = m(x - x_1)$ |
| Example intersection | $y = 2x + 1;\ y = x + 4$ |

For the example, set the expressions equal: x = 3 and y = 7.
A solution must work in both equations.

## Find and count roots

| | |
|---|---|
| Discriminant | $D = b^2 - 4ac$ |
| Roots when D is nonnegative | $x = (-b \pm \sqrt{(D)})/(2a)$ |
| Sum of roots | $r_1 + r_2 = -b/a$ |
| Product of roots | $r_1\, r_2 = c/a$ |

For real coefficients, D > 0 gives two real roots, D = 0 one repeated root, D < 0 no real roots.

## Fractional powers & roots

| | |
|---|---|
| Square root | $a^{1/2} = \sqrt{(a)}$ |
| General positive base | $a^{m/n} = (a^{1/n})^m$ |
| Example | $27^{2/3} = 3^2 = 9$ |
| Square root of a square | $\sqrt{(x^2)} = \lvert x \rvert$ |

For the general rule use a > 0 and positive integer n. An even real root cannot have a negative radicand.

## Rational expressions & equations

Exclude original denominator zeros

$(x^2 - 4)/(x - 2) = x + 2,\ x \neq 2$

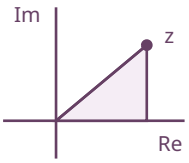
| | |
|---|---|
| Equation example | $6/(x - 1) = 2 \Rightarrow x = 4$ |
| Check candidate | $6/(4 - 1) = 2$ |

Cancel factors after factoring, and keep the restrictions from the original expression.

## Complex numbers

| | |
|---|---|
| Imaginary unit | $i^2 = -1$ |
| Conjugate of a + bi | $a - bi$ |
| Conjugate product | $(a + bi)(a - bi) = a^2 + b^2$ |
| Example | $(2 + 3i)(2 - 3i) = 13$ |

The real part of a + bi is a. The imaginary part is the real coefficient b.




## Compose, invert and transform

| | |
|---|---|
| Composition | $(f \text{ composed with } g)(x) = f(g(x))$ |
| Inverse on one-to-one domain | $g(f(x)) = x$ |
| Transform a parent function | $y = A\, f(B(x - h)) + k$ |

In composition, g(x) must belong to the domain of f.
For B != 0, move right by h and scale horizontal distances by 1/|B|.

## Rates and percent models

| | |
|---|---|
| Distance at a constant rate | $d = rt$ |
| Scale a quantity | $new = old \times (1 + r)$ |
| Find signed change | $r = (new - old)/old$ |
| Example: 80 to 68 | $r = -12/80 = -15\%$ |

For a positive original value, a negative r is a decrease.
Compare units before solving.

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## Coordinate geometry

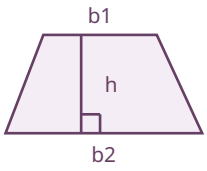
| | |
|---|---|
| Slope | $m = (y_2 - y_1) / (x_2 - x_1)$ |
| Midpoint | $((x_1 + x_2)/2, (y_1 + y_2)/2)$ |
| Squared distance | $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ |

Take the nonnegative square root to get d. A vertical line has no defined slope.

## Area of plane figures

| | |
|---|---|
| Triangle | $A = \frac{1}{2}bh$ |
| Rectangle or parallelogram | $A = bh$ |
| Trapezoid | $A = \frac{1}{2}(b_1 + b_2)h$ |

Measure h perpendicular to the base.
The trapezoid bases are parallel.
Use square units.




## Volume

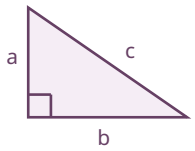
| | |
|---|---|
| Prism or cylinder | $V = Bh$ |
| Pyramid or cone | $V = Bh / 3$ |
| Sphere | $V = (4/3)\pi r^3$ |

B is the area of one base. h is perpendicular height.
Use cubic units.

## Special triangles and angle rules

| | |
|---|---|
| 45°–45°–90° side ratios | $1 : 1 : \sqrt{2}$ |
| 30°–60°–90° side ratios | $1 : \sqrt{3} : 2$ |
| Triangle interior sum | $A + B + C = 180°$ |
| Polygon interior sum | $(n - 2)180°$ |

In a 30°–60°–90° triangle, the shortest side is opposite 30°.




## Right triangle trigonometry

| | |
|---|---|
| Sine | $\sin(\text{angle}) = \text{opposite} / \text{hypotenuse}$ |
| Cosine | $\cos(\text{angle}) = \text{adjacent} / \text{hypotenuse}$ |
| Tangent | $\tan(\text{angle}) = \text{opposite} / \text{adjacent}$ |
| Side lengths | $a^2 + b^2 = c^2$ |

Choose the acute angle before naming the legs. In the side rule, c is the hypotenuse.

## Center and spread

| | |
|---|---|
| Mean | sum / count |
| Median | middle of the ordered data |
| Range | maximum − minimum |

For an even count, the median is the mean of the two middle values.
Example: 1, 4, 7, 8 has median 5.5.

## Basic probability

| | |
|---|---|
| Equally likely outcomes | $P(A) = \text{favorable} / \text{total}$ |
| Complement | $P(\text{not } A) = 1 - P(A)$ |
| Example: fair die | $P(\text{even}) = 3/6 = \frac{1}{2}$ |

The favorable count comes from the group named in the question.

## Matrix operations and shapes

| | |
|---|---|
| Addition | add entries for equal shapes |
| Scalar multiplication | multiply each entry by the scalar |
| Transpose | rows become columns |
| Product transpose | $(AB)^T = B^T A^T$ |

The transpose of an m by n matrix is n by m.