

Uniform Distribution

For X uniform on the stated closed interval [a,b], use density 1/(b-a) on the support and 0 outside it. Clip probability intervals to the support, divide overlap lengths for conditional intervals, include the boundary branches of the CDF, and give exact values where practical; otherwise round the final value to four decimals.

1. Let X be uniform on [-3, -1]. Find the 40th percentile q, defined by $P(X \leq q) = 0.4$.

Given: X is measured in units and is continuous with constant density on [-3, -1] and density 0 outside; endpoints have probability 0.

2. Let X be uniform on [-6, 5]. Find $\text{Var}(X)$. State the squared units.

Given: X is measured in units and is continuous with constant density on [-6, 5] and density 0 outside; endpoints have probability 0.

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3.

Let X be uniform on [-6, 0]. Find $P(-6 \leq X \leq 0 \mid -8 \leq X \leq -1)$.

Given: X is measured in units and is continuous with constant density on [-6, 0] and density 0 outside; endpoints have probability 0.

4.

Let X be uniform on [-4, 6]. Find $P(X \geq 5)$.

Given: X is measured in units and is continuous with constant density on [-4, 6] and density 0 outside; endpoints have probability 0.

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5. Let X be uniform on $[-5, -2]$. Two model descriptions are proposed. Model A has constant PDF $1/3$ on $[-5, -2]$ and 0 outside. Model B assigns probability $d/3$ to every subinterval of length d inside $[-5, -2]$. Select which descriptions represent Uniform $(-5, -2)$, then state the support, full piecewise PDF, and full piecewise CDF.

Given: X is measured in units and is continuous with constant density on $[-5, -2]$ and density 0 outside; endpoints have probability 0.

6. Let X be uniform on $[5, 15]$. Find $P(3 \leq X \leq 17)$.

Given: X is measured in units and is continuous with constant density on $[5, 15]$ and density 0 outside; endpoints have probability 0.

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7. Let X be uniform on $[-2, 1]$. Find $E(X)$.

Given: X is measured in units and is continuous with constant density on $[-2, 1]$ and density 0 outside; endpoints have probability 0.

8. Let X be uniform on $[-2, 2]$. Let X be uniform on $[-2, 2]$. Write the full piecewise CDF, with correct boundary branches.

Given: X is measured in units and is continuous with constant density on $[-2, 2]$ and density 0 outside; endpoints have probability 0.

Uniform Distribution

Worked Answer Key

For X uniform on the stated closed interval [a,b], use density $1/(b-a)$ on the support and 0 outside it. Clip probability intervals to the support, divide overlap lengths for conditional intervals, include the boundary branches of the CDF, and give exact values where practical; otherwise round the final value to four decimals.

1.

Let X be uniform on [-3, -1]. Find the 40th percentile q, defined by $P(X \leq q) = 0.4$.

Answer: q = -2.2 units

1. For a uniform distribution, $q = a + p(b-a)$.

2. $q = -3 + 0.4(2) = -2.2$ units.

2.

Let X be uniform on [-6, 5]. Find $\text{Var}(X)$. State the squared units.

Answer: 121/12 squared units (approximately 10.0833)

1. For $\text{Uniform}(a,b)$, $\text{Var}(X) = (b-a)^2/12$.

2. $\text{Var}(X) = (5+6)^2/12 = 121/12$ squared units.

Uniform Distribution

Worked Answer Key

For X uniform on the stated closed interval [a,b], use density 1/(b-a) on the support and 0 outside it. Clip probability intervals to the support, divide overlap lengths for conditional intervals, include the boundary branches of the CDF, and give exact values where practical; otherwise round the final value to four decimals.

3. Let X be uniform on [-6, 0]. Find $P(-6 \leq X \leq 0 \mid -8 \leq X \leq -1)$.

Answer: 1 (decimal 1.0000)

- The conditioning interval overlaps the support in a length of 5, which is positive.
- The event-and-condition intersection overlaps the support in a length of 5.
- Conditional probability = $5/5 = 1$.

4. Let X be uniform on [-4, 6]. Find $P(X \geq 5)$.

Answer: 1/10 (decimal 0.1000)

- The favorable length is $6-5=1$.
- $P(X \geq 5) = (6-5)/10 = 1/10$.

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5. Let X be uniform on $[-5, -2]$. Two model descriptions are proposed. Model A has constant PDF $1/3$ on $[-5, -2]$ and 0 outside. Model B assigns probability $d/3$ to every subinterval of length d inside $[-5, -2]$. Select which descriptions represent Uniform $(-5, -2)$, then state the support, full piecewise PDF, and full piecewise CDF.

Answer: Both models; support $[-5, -2]$; $f(x) = 1/3$ for $-5 \leq x \leq -2$, and 0 otherwise; $F(x)=0$ for $x < -5$, $((x + 5))/3$ for $-5 < x < -2$, and 1 for $x \geq -2$.

1. The support length is $-2+5=3$.
2. A constant density of $1/3$ gives area $d/3$ over every internal interval of length d , so Models A and B are equivalent.
3. The CDF accumulates length from -5 : 0 below the support, $((x + 5))/3$ inside, and 1 above.

6. Let X be uniform on $[5, 15]$. Find $P(3 \leq X \leq 17)$.

Answer: 1 (decimal 1.0000)

1. Clip the requested interval to the support $[5, 15]$.
2. The overlap is $[5, 15]$, with length 10.
3. Probability = overlap length / support length = $10/10 = 1$.

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7. Let X be uniform on $[-2, 1]$. Find $E(X)$.

Answer: -1/2 units (approximately -0.5000)

1. The mean of a uniform interval is its midpoint.
2. $E(X) = (-2+1)/2 = -1/2$ units.

8. Let X be uniform on $[-2, 2]$. Let X be uniform on $[-2, 2]$. Write the full piecewise CDF, with correct boundary branches.

Answer: $F(x) = 0$ for $x \leq -2$; $F(x) = (x + 2)/4$ for $-2 < x < 2$; $F(x) = 1$ for $x \geq 2$.

1. $F(x) = 0$ at and below -2.
2. Within the support, accumulated length is $(x + 2)$, divided by total length 4.
3. $F(x) = 1$ at and above 2.