

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Sampling Distributions and the CLT

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

1. An iid sample has $n=49$, population mean 10, and population variance 16. The population is normal, so the stated normal sampling law is exact. Explain what the LLN and CLT each say here without claiming the approximation is universally exact.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

2. An iid sample has $n=36$, population mean 11, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $P(\bar{X} \leq 11.0000)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(0.0000)=0.499999985$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Sampling Distributions and the CLT

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

- 3. An iid sample has $n=36$, population mean 10, and population variance 36. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $P(9.0000 < \bar{X} < 12.0000)$.**

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(-1.0000)=0.158655261$; $\Phi(2.0000)=0.977249870$.

- 4. An iid sample has $n=81$, population mean 20, and population variance 16. The population is normal, so the stated normal sampling law is exact. Find $P(1584.0000 < S < 1674.0000)$.**

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(-1.0000)=0.158655261$; $\Phi(1.5000)=0.933192798$.

Name _____ Score _____ Teacher _____ Date _____

Sampling Distributions and the CLT

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

5. An iid sample has $n=81$, population mean 17, and population variance 16. The population is normal, so the stated normal sampling law is exact. Find $E(\bar{X})$, $\text{Var}(\bar{X})$, and standard error.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

6. An iid sample has $n=16$, population mean 16, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $E(\bar{S})$, $\text{Var}(\bar{S})$, and standard error.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Sampling Distributions and the CLT

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

7. For X binomial with $n=179$ and $p=0.4$, use a normal approximation with continuity correction to estimate $P(X \leq 74)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(0.4425)=0.670936300$. This continuity-corrected normal result is approximate.

8. An iid sample has $n=64$, population mean 14, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . If sample size changes from 64 to 100, compare the standard errors of $\bar{X}$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Name _____ Score _____ Teacher _____ Date _____

Sampling Distributions and the CLT

Worked Answer Key

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

1. An iid sample has $n=49$, population mean 10, and population variance 16. The population is normal, so the stated normal sampling law is exact. Explain what the LLN and CLT each say here without claiming the approximation is universally exact.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Answer: LLN concerns concentration; CLT concerns standardized shape

1. LLN: as n grows, $\bar{X}$ concentrates near μ because $\text{Var}(\bar{X}) = \sigma^2/n$.
2. CLT: under iid finite-variance assumptions, the standardized mean approaches a normal distribution; $n=30$ does not guarantee accuracy.

2. An iid sample has $n=36$, population mean 11, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $P(\bar{X} \leq 11.0000)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(0.0000) = 0.499999985$.

Answer: approximately 0.5000

1. The printed cutoff standardizes to $z=0.0000$.
2. Use $\Phi(z)$: approximately 0.5000.

Sampling Distributions and the CLT

Worked Answer Key

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

3. An iid sample has $n=36$, population mean 10, and population variance 36. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $P(9.0000 < \bar{X} < 12.0000)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(-1.0000)=0.158655261$; $\Phi(2.0000)=0.977249870$.

Answer: approximately 0.8186

1. Use the printed cutoffs: $z=-1.0000$ and 2.0000 .
2. Subtract the supplied CDF values: approximately 0.8186.

4. An iid sample has $n=81$, population mean 20, and population variance 16. The population is normal, so the stated normal sampling law is exact. Find $P(1584.0000 < S < 1674.0000)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(-1.0000)=0.158655261$; $\Phi(1.5000)=0.933192798$.

Answer: approximately 0.7745

1. Use the printed cutoffs: $z=-1.0000$ and 1.5000 .
2. Subtract the supplied CDF values: approximately 0.7745.

Sampling Distributions and the CLT

Worked Answer Key

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

5. An iid sample has $n=81$, population mean 17, and population variance 16. The population is normal, so the stated normal sampling law is exact. Find $E(\bar{X})$, $\text{Var}(\bar{X})$, and standard error.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Answer: mean 17.0000, variance 0.1975, SE 0.4444

1. $E(\bar{X})=\mu$, $\text{Var}(\bar{X})=\sigma^2/n$, $\text{SE}=\sigma/\sqrt{n}$.
2. mean 17.0000, variance 0.1975, SE 0.4444

6. An iid sample has $n=16$, population mean 16, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . Find $E(S)$, $\text{Var}(S)$, and standard error.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Answer: mean 256.0000, variance 256.0000, SE 16.0000

1. $E(S)=n\mu$, $\text{Var}(S)=n\sigma^2$, $\text{SE}=\sigma\sqrt{n}$.
2. mean 256.0000, variance 256.0000, SE 16.0000

Sampling Distributions and the CLT

Worked Answer Key

Assume iid observations with the stated finite mean and variance. Label exact normal results versus CLT approximations; use provided normal-CDF values and do not treat a sample-size rule as a guarantee.

7. For X binomial with $n=179$ and $p=0.4$, use a normal approximation with continuity correction to estimate $P(X \leq 74)$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Provided normal CDF: $\Phi(0.4425)=0.670936300$. This continuity-corrected normal result is approximate.

Answer: approximately 0.6709

1. Replace $X \leq 74$ by the continuous cutoff 74.5.
2. The printed $z=0.4425$ gives approximately 0.6709.

8. An iid sample has $n=64$, population mean 14, and population variance 16. The population is not stated normal; use the requested CLT approximation, whose accuracy depends on shape and n . If sample size changes from 64 to 100, compare the standard errors of $\bar{X}$.

Assumptions: Sampling is iid with finite variance; sums and means use the same n observations.

Use the stated iid assumptions and sampling-moment formulas.

Answer: SE changes from 0.5000 to 0.4000

1. $SE(\bar{X}) = \sigma / \sqrt{n}$.
2. SE changes from 0.5000 to 0.4000