

Poisson Distribution

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

1. Arrivals occur independently at a constant average rate of 0.4 per hour. Observe 0.75 hours. Find the probability of at least 6 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

2. Arrivals occur independently at a constant average rate of 0.4 per hour. Observe 1.5 hours. Find the probability of exactly 7 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Poisson Distribution

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

3. Arrivals occur independently at a constant average rate of 0.25 per hour. Observe 2 hours. Find the mean and population variance of the number of arrivals. State the count support and distinguish this Poisson count from an exponential waiting time.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

4. Surface flaws occur independently at a constant average rate of 1.25 per kilometer. Observe 0.5 kilometers. Find the probability of at most 7 surface flaws.

Assumptions: The surface flaws occur independently with the same constant average rate throughout the stated kilometer exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Poisson Distribution

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

5. Surface flaws occur independently at a constant average rate of 1.5 per kilometer. Observe 2 kilometers. Find the probability of exactly 6 surface flaws.

Assumptions: The surface flaws occur independently with the same constant average rate throughout the stated kilometer exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

6. Material defects occur independently at a constant average rate of 3 per square meter. Observe 4 square meters. Find the mean and population variance of the number of material defects. State the count support and distinguish this Poisson count from an exponential waiting time.

Assumptions: The material defects occur independently with the same constant average rate throughout the stated square meter exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Poisson Distribution

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

7. Arrivals occur independently at a constant average rate of 1 per hour. Observe 2 hours. Find the probability of at least 0 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

8. Arrivals occur independently at a constant average rate of 2.5 per hour. Observe 1.5 hours. Find the probability of at most 1 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Poisson Distribution
Worked Answer Key

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

1. Arrivals occur independently at a constant average rate of 0.4 per hour. Observe 0.75 hours. Find the probability of at least 6 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 7.835e-7

1. $\lambda = (0.4)(0.75) = 0.3$.

2. $P(X \geq 6) = 1 - P(X \leq 5) = 7.835e-7$.

2. Arrivals occur independently at a constant average rate of 0.4 per hour. Observe 1.5 hours. Find the probability of exactly 7 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 3.048e-6

1. $\lambda = (0.4)(1.5) = 0.6$ arrivals per observed exposure.

2. $P(X=7) = \exp(-0.6)(0.6)^7/7! = 3.048e-6$.

Poisson Distribution

Worked Answer Key

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

- 3. Arrivals occur independently at a constant average rate of 0.25 per hour. Observe 2 hours. Find the mean and population variance of the number of arrivals. State the count support and distinguish this Poisson count from an exponential waiting time.**

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: mean 0.5, variance 0.5; support {0,1,...}; Poisson models a count, while exponential models a waiting time

1. $\lambda = (0.25)(2) = 0.5$.
2. For a Poisson count, $E(X) = \lambda = 0.5$ and $\text{Var}(X) = \lambda = 0.5$.
3. Its support is $\{0,1,\dots\}$. Poisson models the count in a fixed exposure; an exponential variable models a nonnegative waiting time between events.

- 4. Surface flaws occur independently at a constant average rate of 1.25 per kilometer. Observe 0.5 kilometers. Find the probability of at most 7 surface flaws.**

Assumptions: The surface flaws occur independently with the same constant average rate throughout the stated kilometer exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 1.000

1. $\lambda = (1.25)(0.5) = 0.625$.
2. $P(X \leq 7) = \sum_{x=0}^7 \exp(-\lambda) \lambda^x / x! = 1.000$.

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Poisson Distribution

Worked Answer Key

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

5. Surface flaws occur independently at a constant average rate of 1.5 per kilometer. Observe 2 kilometers. Find the probability of exactly 6 surface flaws.

Assumptions: The surface flaws occur independently with the same constant average rate throughout the stated kilometer exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 0.050

1. $\lambda = (1.5)(2) = 3$ surface flaws per observed exposure.
2. $P(X=6) = \exp(-3)(3)^6/6! = 0.050$.

6. Material defects occur independently at a constant average rate of 3 per square meter. Observe 4 square meters. Find the mean and population variance of the number of material defects. State the count support and distinguish this Poisson count from an exponential waiting time.

Assumptions: The material defects occur independently with the same constant average rate throughout the stated square meter exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: mean 12, variance 12; support {0,1,...}; Poisson models a count, while exponential models a waiting time

1. $\lambda = (3)(4) = 12$.
2. For a Poisson count, $E(X) = \lambda = 12$ and $\text{Var}(X) = \lambda = 12$.
3. Its support is $\{0,1,\dots\}$. Poisson models the count in a fixed exposure; an exponential variable models a nonnegative waiting time between events.

Poisson Distribution

Worked Answer Key

Assume events occur independently at a constant average rate. Convert rate and exposure to lambda in matching units. Use a calculator for exponentials and Poisson sums. Round final probabilities to the 3 decimal places; if that would display 0, use scientific notation.

7. Arrivals occur independently at a constant average rate of 1 per hour. Observe 2 hours. Find the probability of at least 0 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 1.000

1. $\lambda = (1)(2) = 2$.
2. Every Poisson count is at least 0, so the probability is 1.

8. Arrivals occur independently at a constant average rate of 2.5 per hour. Observe 1.5 hours. Find the probability of at most 1 arrivals.

Assumptions: The arrivals occur independently with the same constant average rate throughout the stated hour exposure; disjoint exposure segments have independent counts.

Reference: $\lambda = (\text{rate per unit})(\text{exposure})$, and $P(X=k) = \exp(-\lambda) \lambda^k / k!$. Calculator required for exponential values and Poisson sums.

Answer: approximately 0.112

1. $\lambda = (2.5)(1.5) = 3.75$.
2. $P(X \leq 1) = \text{sum from } x=0 \text{ through } 1 \text{ of } \exp(-\lambda) \lambda^x / x! = 0.112$.