

Joint Continuous Distributions

Use the stated rectangular or triangular support. Integrate density over area; support restrictions are part of every marginal, conditional, and independence argument.

1. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 3.2, 0\leq y\leq 12.7\}$ and 0 elsewhere. Find $E(X)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

2. The joint density is constant c on $S=\{(x,y): x\geq 0, y\geq 0, x+y\leq 6.1\}$ and 0 elsewhere. Find the full marginal densities f_X and f_Y , including support.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
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3. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 4.4, 0\leq y\leq 11.5\}$ and 0 elsewhere. Find $P(X\leq 2.2, Y\leq 5.75)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

4. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 8.4, 0\leq y\leq 7.5\}$ and 0 elsewhere. Find c .

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

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5. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 8.2, 0\leq y\leq 7.7\}$ and 0 elsewhere. Find $F(4.1,3.85)=P(X\leq 4.1,Y\leq 3.85)$

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

6. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 2.4, 0\leq y\leq 13.5\}$ and 0 elsewhere. Find the full conditional density $f(Y | X=1.2)$, including values outside conditional support.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

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7. The joint density is constant c on $S=\{(x,y): x\geq 0, y\geq 0, x+y\leq 12.1\}$ and 0 elsewhere. Are X and Y independent? Use density and support reasoning.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

8. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 13, 0\leq y\leq 2.9\}$ and 0 elsewhere. Find $P(X\leq 6.5, Y\leq 1.45)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Joint Continuous Distributions

Worked Answer Key

Use the stated rectangular or triangular support. Integrate density over area; support restrictions are part of every marginal, conditional, and independence argument.

1. The joint density is constant c on $S=\{(x,y): 0 \leq x \leq 3.2, 0 \leq y \leq 12.7\}$ and 0 elsewhere. Find $E(X)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: $E(X)=1.6000$

1. The uniform rectangular marginal is uniform on $[0,3.2]$, so $E(X)=1.6000$.

2. The joint density is constant c on $S=\{(x,y): x \geq 0, y \geq 0, x+y \leq 6.1\}$ and 0 elsewhere. Find the full marginal densities f_X and f_Y , including support.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: $f_X(x)=2(6.1-x)/37.21$ for $0 \leq x \leq 6.1$, 0 otherwise; $f_Y(y)=2(6.1-y)/37.21$ for $0 \leq y \leq 6.1$, 0 otherwise

1. Integrate the joint density over the support allowed for the other variable.
2. $f_X(x)=2(6.1-x)/37.21$ for $0 \leq x \leq 6.1$, 0 otherwise; $f_Y(y)=2(6.1-y)/37.21$ for $0 \leq y \leq 6.1$, 0 otherwise

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Joint Continuous Distributions

Worked Answer Key

Use the stated rectangular or triangular support. Integrate density over area; support restrictions are part of every marginal, conditional, and independence argument.

3. The joint density is constant c on $S=\{(x,y): 0 \leq x \leq 4.4, 0 \leq y \leq 11.5\}$ and 0 elsewhere. Find $P(X \leq 2.2, Y \leq 5.75)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: 0.2500

1. Probability is density times the requested rectangular area: 0.2500.

4. The joint density is constant c on $S=\{(x,y): 0 \leq x \leq 8.4, 0 \leq y \leq 7.5\}$ and 0 elsewhere. Find c .

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: $c=0.0159$

1. Support area is $8.4(7.5)=63$; $c=1/\text{area}=0.0159$.

Joint Continuous Distributions

Worked Answer Key

Use the stated rectangular or triangular support. Integrate density over area; support restrictions are part of every marginal, conditional, and independence argument.

5. The joint density is constant c on $S=\{(x,y): 0 \leq x \leq 8.2, 0 \leq y \leq 7.7\}$ and 0 elsewhere. Find $F(4.1, 3.85) = P(X \leq 4.1, Y \leq 3.85)$

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: 0.2500

1. Integrate the density over the support portion below both cutoffs: 0.2500.

6. The joint density is constant c on $S=\{(x,y): 0 \leq x \leq 2.4, 0 \leq y \leq 13.5\}$ and 0 elsewhere. Find the full conditional density $f(Y | X=1.2)$, including values outside conditional support.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.

Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: 0.0741 for $0 \leq y \leq 13.5$, and 0 otherwise

1. Divide $f(x,y)$ by $f_X(x)$ on the conditional support.
2. 0.0741 for $0 \leq y \leq 13.5$, and 0 otherwise

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Use the stated rectangular or triangular support. Integrate density over area; support restrictions are part of every marginal, conditional, and independence argument.

7. The joint density is constant c on $S=\{(x,y): x\geq 0, y\geq 0, x+y\leq 12.1\}$ and 0 elsewhere. Are X and Y independent? Use density and support reasoning.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: not independent

1. The triangular support is not a Cartesian product: allowed y values depend on x , so X and Y are not independent.

8. The joint density is constant c on $S=\{(x,y): 0\leq x\leq 13, 0\leq y\leq 2.9\}$ and 0 elsewhere. Find $P(X\leq 6.5, Y\leq 1.45)$.

Assumptions: The density is zero outside the explicitly stated support; all integrals use the shown coordinate units.
Probability is a double integral of density over support. A point has probability 0; conditional and marginal densities include their support restrictions.

Answer: 0.2500

1. Probability is density times the requested rectangular area: 0.2500.