

Name _____ Score _____ Teacher _____ Date _____

Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

- 1. The same test has sensitivity 0.9 and false-positive rate 0.2. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.1 and 0.3.**

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / [(\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate}]$.

- 2. A condition has prior 0.1. A test has $P(\text{positive} \mid \text{condition}) = 0.75$ and $P(\text{positive} \mid \text{no condition}) = 0.1$. A student claims $P(\text{condition} \mid \text{positive}) = 0.75$. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

3. A 96-person table has 48 in A, 24 in B, and 12 in both A and B. Complete the four cell counts and decide whether A and B are independent.

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

4. The same test has sensitivity 0.8 and false-positive rate 0.2. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.1 and 0.3.

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / [(\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate}]$.

Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

5. A 112-person table has 56 in A, 28 in B, and 14 in both A and B. Complete the four cell counts and decide whether A and B are independent.

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

6. A condition has prior 0.1. A test has $P(\text{positive} \mid \text{condition}) = 0.75$ and $P(\text{positive} \mid \text{no condition}) = 0.05$. A student claims $P(\text{condition} \mid \text{positive}) = 0.75$. Explain the error and find the correct posterior.

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

7. A 128-person table has 64 in A, 32 in B, and 16 in both A and B. Complete the four cell counts and decide whether A and B are independent.

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

8. The same test has sensitivity 0.8 and false-positive rate 0.1. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.01 and 0.4.

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / [(\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate}]$.

Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

- 1. The same test has sensitivity 0.9 and false-positive rate 0.2. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.1 and 0.3.**

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / ((\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate})$.

Answer: 0.333 versus 0.659

1. For prior 0.1, $\text{posterior} = 0.333$.
2. For prior 0.3, $\text{posterior} = 0.659$.
3. The likelihoods are identical; the larger prior produces the larger posterior.

- 2. A condition has prior 0.1. A test has $P(\text{positive} \mid \text{condition}) = 0.75$ and $P(\text{positive} \mid \text{no condition}) = 0.1$. A student claims $P(\text{condition} \mid \text{positive}) = 0.75$. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Answer: the claim reverses the conditional; posterior 0.455

1. $P(\text{positive}) = 0.1(0.75) + [1 - 0.1](0.1)$.
2. $P(\text{condition} \mid \text{positive}) = 0.455$, which need not equal $P(\text{positive} \mid \text{condition})$.

Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

- 3. A 96-person table has 48 in A, 24 in B, and 12 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

Answer: independent

1. The cells are both=12, A only=36, B only=12, neither=36.
2. $P(A \text{ intersection } B) = 12/96 = 0.125$; $P(A)P(B) = (48/96)(24/96) = 0.125$.
3. The values agree, so the events are independent.

- 4. The same test has sensitivity 0.8 and false-positive rate 0.2. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.1 and 0.3.**

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / [(\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate}]$.

Answer: 0.308 versus 0.632

1. For prior 0.1, posterior=0.308.
2. For prior 0.3, posterior=0.632.
3. The likelihoods are identical; the larger prior produces the larger posterior.

Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

5. A 112-person table has 56 in A, 28 in B, and 14 in both A and B. Complete the four cell counts and decide whether A and B are independent.

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

Answer: independent

1. The cells are both=14, A only=42, B only=14, neither=42.
2. $P(A \text{ intersection } B) = 14/112 = 0.125$; $P(A)P(B) = (56/112)(28/112) = 0.125$.
3. The values agree, so the events are independent.

6. A condition has prior 0.1. A test has $P(\text{positive} \mid \text{condition}) = 0.75$ and $P(\text{positive} \mid \text{no condition}) = 0.05$. A student claims $P(\text{condition} \mid \text{positive}) = 0.75$. Explain the error and find the correct posterior.

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Answer: the claim reverses the conditional; posterior 0.625

1. $P(\text{positive}) = 0.1(0.75) + [1 - 0.1](0.05)$.
2. $P(\text{condition} \mid \text{positive}) = 0.625$, which need not equal $P(\text{positive} \mid \text{condition})$.

Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

7. A 128-person table has 64 in A, 32 in B, and 16 in both A and B. Complete the four cell counts and decide whether A and B are independent.

Reference: Events are independent exactly when $P(A \text{ intersection } B) = P(A)P(B)$.

Answer: independent

- 1. The cells are both=16, A only=48, B only=16, neither=48.
- 2. $P(A \text{ intersection } B) = 16/128 = 0.125$; $P(A)P(B) = (64/128)(32/128) = 0.125$.
- 3. The values agree, so the events are independent.

8. The same test has sensitivity 0.8 and false-positive rate 0.1. Compare $P(\text{condition} \mid \text{positive})$ in populations with priors 0.01 and 0.4.

Reference: Bayes: $\text{posterior} = (\text{prior} \times \text{sensitivity}) / ((\text{prior} \times \text{sensitivity}) + (1 - \text{prior}) \times \text{false-positive rate})$.

Answer: 0.075 versus 0.842

- 1. For prior 0.01, posterior=0.075.
- 2. For prior 0.4, posterior=0.842.
- 3. The likelihoods are identical; the larger prior produces the larger posterior.