Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

1. **The same test has sensitivity 0.9 and false-positive rate 0.2. Compare P(condition | positive) in populations with priors 0.1 and 0.3.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

2. **A condition has prior 0.1. A test has P(positive | condition)=0.75 and P(positive | no condition)=0.1. A student claims P(condition | positive)=0.75. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

State the model or rule, show exact setup, then round only the final decimal.

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Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

3. **A 96-person table has 48 in A, 24 in B, and 12 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

4. **The same test has sensitivity 0.8 and false-positive rate 0.2. Compare P(condition | positive) in populations with priors 0.1 and 0.3.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

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5. **A 112-person table has 56 in A, 28 in B, and 14 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

6. **A condition has prior 0.1. A test has P(positive | condition)=0.75 and P(positive | no condition)=0.05. A student claims P(condition | positive)=0.75. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

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Independence and Base Rates

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

7. **A 128-person table has 64 in A, 32 in B, and 16 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

8. **The same test has sensitivity 0.8 and false-positive rate 0.1. Compare P(condition | positive) in populations with priors 0.01 and 0.4.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

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Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

1. **The same test has sensitivity 0.9 and false-positive rate 0.2. Compare P(condition | positive) in populations with priors 0.1 and 0.3.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

Answer: 0.333 versus 0.659

1. For prior 0.1, posterior=0.333.
2. For prior 0.3, posterior=0.659.
3. The likelihoods are identical; the larger prior produces the larger posterior.

2. **A condition has prior 0.1. A test has P(positive | condition)=0.75 and P(positive | no condition)=0.1. A student claims P(condition | positive)=0.75. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Answer: the claim reverses the conditional; posterior 0.455

1. P(positive)=0.1(0.75)+[1-0.1](0.1).
2. P(condition | positive)=0.455, which need not equal P(positive | condition).

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Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

3. **A 96-person table has 48 in A, 24 in B, and 12 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

Answer: independent

1. The cells are both=12, A only=36, B only=12, neither=36.
2. P(A intersection B)=12/96=0.125; P(A)P(B)=(48/96)(24/96)=0.125.
3. The values agree, so the events are independent.

4. **The same test has sensitivity 0.8 and false-positive rate 0.2. Compare P(condition | positive) in populations with priors 0.1 and 0.3.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

Answer: 0.308 versus 0.632

1. For prior 0.1, posterior=0.308.
2. For prior 0.3, posterior=0.632.
3. The likelihoods are identical; the larger prior produces the larger posterior.

Name _____ Score _____ Teacher _____ Date _____

Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

5. **A 112-person table has 56 in A, 28 in B, and 14 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

Answer: independent

1. The cells are both=14, A only=42, B only=14, neither=42.
2. P(A intersection B)=14/112=0.125; P(A)P(B)=(56/112)(28/112)=0.125.
3. The values agree, so the events are independent.

6. **A condition has prior 0.1. A test has P(positive | condition)=0.75 and P(positive | no condition)=0.05. A student claims P(condition | positive)=0.75. Explain the error and find the correct posterior.**

Reference: Reversing a conditional changes its denominator. Use the prior and total positive probability.

Answer: the claim reverses the conditional; posterior 0.625

1. P(positive)=0.1(0.75)+[1-0.1](0.05).
2. P(condition | positive)=0.625, which need not equal P(positive | condition).

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Independence and Base Rates

Worked Answer Key

Use joint and marginal probabilities to check independence. In Bayes problems, keep the prior visible.

7. **A 128-person table has 64 in A, 32 in B, and 16 in both A and B. Complete the four cell counts and decide whether A and B are independent.**

Reference: Events are independent exactly when P(A intersection B)=P(A)P(B).

Answer: independent

1. The cells are both=16, A only=48, B only=16, neither=48.
2. P(A intersection B)=16/128=0.125; P(A)P(B)=(64/128)(32/128)=0.125.
3. The values agree, so the events are independent.

8. **The same test has sensitivity 0.8 and false-positive rate 0.1. Compare P(condition | positive) in populations with priors 0.01 and 0.4.**

Reference: Bayes: posterior=(prior x sensitivity)/[(prior x sensitivity)+(1-prior)x false-positive rate].

Answer: 0.075 versus 0.842

1. For prior 0.01, posterior=0.075.
2. For prior 0.4, posterior=0.842.
3. The likelihoods are identical; the larger prior produces the larger posterior.