

Discrete Probability Distributions

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

1. Find the expected value E(X).

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	23	25	27	29	31
P(X=x)	1/10	1/10	3/20	1/4	2/5

2. Find the population variance Var(X).

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	15	17	19
P(X=x)	1/5	7/20	9/20

Discrete Probability Distributions

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

3. Find the missing probability $P(X = 59)$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	57	59	61
$P(X=x)$	$\frac{1}{5}$	?	$\frac{9}{20}$

4. Find $P(X \text{ is one of } \{60, 64\})$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	60	62	64
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

Discrete Probability Distributions

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

5. Find $P(X \text{ is one of } \{46\})$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	45	46	47
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

6. Find the expected value $E(X)$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	78	79	80
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

Discrete Probability Distributions

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

7. Find the missing probability $P(X = -11)$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	-12	-11	-10
$P(X=x)$	$\frac{1}{5}$	?	$\frac{9}{20}$

8. Find the population variance $\text{Var}(X)$.

Assumptions: X has exactly the finite integer support shown; every listed probability is nonnegative and the complete table totals 1.

x	44	45	46	47	48
$P(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{2}{5}$

Discrete Probability Distributions

Worked Answer Key

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

1. Find the expected value $E(X)$.

x	23	25	27	29	31
$P(X=x)$	1/10	1/10	3/20	1/4	2/5

Answer: $57/2$

- 1. $E(X)$ = sum of $xP(X=x)$.
- 2. $E(X)$ = $(23(1/10) + 25(1/10) + 27(3/20) + 29(1/4) + 31(2/5)) = 57/2$.

2. Find the population variance $Var(X)$.

x	15	17	19
$P(X=x)$	1/5	7/20	9/20

Answer: $47/20$

- 1. $E(X)$ = $35/2$ and $E(X^2)$ = $1543/5$.
- 2. $Var(X)$ = $E(X^2) - [E(X)]^2 = 47/20$.

Discrete Probability Distributions

Worked Answer Key

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

3. Find the missing probability $P(X = 59)$.

x	57	59	61
$P(X=x)$	$\frac{1}{5}$	?	$\frac{9}{20}$

Answer: $\frac{7}{20}$

- 1. A valid distribution has total probability 1.
- 2. Missing probability = $1 - (\frac{1}{5} + \frac{9}{20}) = \frac{7}{20}$.

4. Find $P(X \text{ is one of } \{60, 64\})$.

x	60	62	64
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

Answer: $\frac{13}{20}$

- 1. Add the table entries for the stated event.
- 2. $P(\text{event}) = \frac{1}{5} + \frac{9}{20} = \frac{13}{20}$.

Discrete Probability Distributions

Worked Answer Key

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

5. Find $P(X \text{ is one of } \{46\})$.

x	45	46	47
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

Answer: $\frac{7}{20}$

- 1. Add the table entries for the stated event.
- 2. $P(\text{event}) = \frac{7}{20} = \frac{7}{20}$.

6. Find the expected value $E(X)$.

x	78	79	80
$P(X=x)$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{9}{20}$

Answer: $\frac{317}{4}$

- 1. $E(X) = \text{sum of } xP(X=x)$.
- 2. $E(X) = (78(\frac{1}{5}) + 79(\frac{7}{20}) + 80(\frac{9}{20})) = \frac{317}{4}$.

Discrete Probability Distributions

Worked Answer Key

Use the complete labeled probability table. Check that probabilities sum to 1, and show exact fraction work for events, expectation, and population variance.

7. Find the missing probability $P(X = -11)$.

x	-12	-11	-10
$P(X=x)$	$\frac{1}{5}$	?	$\frac{9}{20}$

Answer: $\frac{7}{20}$

1. A valid distribution has total probability 1.
2. Missing probability = $1 - (\frac{1}{5} + \frac{9}{20}) = \frac{7}{20}$.

8. Find the population variance $\text{Var}(X)$.

x	44	45	46	47	48
$P(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{2}{5}$

Answer: $\frac{143}{80}$

1. $E(X) = \frac{187}{4}$ and $E(X^2) = \frac{43747}{20}$.
2. $\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{143}{80}$.