

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

- 1. X has support -3, -1, 1, 3 with probabilities 0.2, 0.3, 0.1, 0.4. For $Y=X^2$, give the PMF and merge repeated transformed values.**

Reference: When several X values map to one Y value, add their probabilities.

- 2. X has mean 10 and variance 7. Let $Y=-3X+3$. Find $E(Y)$, $\text{Var}(Y)$, and $\text{SD}(Y)$.**

Reference: $E(aX+b)=aE(X)+b$; $\text{Var}(aX+b)=a^2 \text{Var}(X)$; SD scales by $|a|$.

Discrete CDFs, Transformations, and Moments

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

3. X is equally likely to be 6 or 10; Y is equally likely to be 4 or 12. Compare their means and standard deviations.

Reference: Equal means do not imply equal spread. Compute probability-weighted variance around the common mean.

4. For support 7, 9, 11, 13 with probabilities 0.25, 0.25, 0.25, 0.25, evaluate $F(6)$, $F(10)$, and $F(14)$.

Reference: A discrete CDF is right-continuous and constant between support points.

Discrete CDFs, Transformations, and Moments

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

5. X has support 8, 10, 12, 14, 16 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3 in matching order. Make a PMF table and a step-CDF table.

Reference: $F(x)=P(X\leq x)$. The CDF is 0 below the support and 1 at or above its largest value.

6. Independent X and Y have $E(X)=11.6$, $\text{Var}(X)=2.44$, $E(Y)=2$, and $\text{Var}(Y)=2$. Find the mean, variance, and SD of $X+Y$.

Reference: Means always add. Variances add here because independence makes covariance zero.

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Discrete CDFs, Transformations, and Moments

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

7. **X has support 10, 12, 14, 16 with probabilities 0.25, 0.25, 0.25, 0.25. Find $E(X)$, $E(X^2)$, $\text{Var}(X)$, and $\text{SD}(X)$.**

Reference: Use probability-weighted sums; $\text{Var}(X)=E(X^2)-[E(X)]^2$ and SD is the positive square root.

8. **X has support 11, 13, 15, 17, 19 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3. Using $q_p=\min\{x \text{ in support: } F(x)\geq p\}$, find $q_{0.7}$.**

Reference: Choose the first support point whose cumulative probability reaches or exceeds p .

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

1. X has support -3, -1, 1, 3 with probabilities 0.2, 0.3, 0.1, 0.4. For $Y=X^2$, give the PMF and merge repeated transformed values.

Reference: When several X values map to one Y value, add their probabilities.

Answer: support 1, 9; probabilities 0.4, 0.6

1. Both -1 and 1 map to 1, so $P(Y=1)=0.3+0.1=0.4$.

2. The outer pair maps to 9, so its probability is $0.2+0.4=0.6$.

2. X has mean 10 and variance 7. Let $Y=-3X+3$. Find $E(Y)$, $\text{Var}(Y)$, and $\text{SD}(Y)$.

Reference: $E(aX+b)=aE(X)+b$; $\text{Var}(aX+b)=a^2 \text{Var}(X)$; SD scales by $|a|$.

Answer: E=-27, Var=63, SD=7.937

1. $E(Y)=-3(10)+3=-27$.

2. $\text{Var}(Y)=(-3)^2(7)=63$; $\text{SD}(Y)=7.937$.

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

3. X is equally likely to be 6 or 10; Y is equally likely to be 4 or 12. Compare their means and standard deviations.

Reference: Equal means do not imply equal spread. Compute probability-weighted variance around the common mean.

Answer: both means 8; SD(X)=2, SD(Y)=4

- 1. Both symmetric pairs have mean 8.
- 2. SD(X)=2 and SD(Y)=4, so Y is more spread out.

4. For support 7, 9, 11, 13 with probabilities 0.25, 0.25, 0.25, 0.25, evaluate F(6), F(10), and F(14).

Reference: A discrete CDF is right-continuous and constant between support points.

Answer: 0, 0.5, 1

- 1. F(6)=0 because it lies below the support.
- 2. F(10)=0.5 because only 7 and 9 are at most 10.
- 3. F(14)=1.

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

5. X has support 8, 10, 12, 14, 16 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3 in matching order. Make a PMF table and a step-CDF table.

Reference: $F(x)=P(X\leq x)$. The CDF is 0 below the support and 1 at or above its largest value.

Answer: CDF values 0.1, 0.25, 0.45, 0.7, 1

1. Cumulative sums at the support are 0.1, 0.25, 0.45, 0.7, 1.
2. Thus $F(x)=0$ below 8, takes those cumulative values across successive steps, and equals 1 from 16 onward.

6. Independent X and Y have $E(X)=11.6$, $\text{Var}(X)=2.44$, $E(Y)=2$, and $\text{Var}(Y)=2$. Find the mean, variance, and SD of $X+Y$.

Reference: Means always add. Variances add here because independence makes covariance zero.

Answer: mean 13.6, variance 4.44, SD 2.107

1. $E(X+Y)=11.6+2=13.6$.
2. $\text{Var}(X+Y)=2.44+2=4.44$; $\text{SD}=2.107$.

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

7. X has support 10, 12, 14, 16 with probabilities 0.25, 0.25, 0.25, 0.25. Find $E(X)$, $E(X^2)$, $\text{Var}(X)$, and $\text{SD}(X)$.

Reference: Use probability-weighted sums; $\text{Var}(X)=E(X^2)-[E(X)]^2$ and SD is the positive square root.

Answer: $E=13$, $E(X^2)=174$, $\text{Var}=5$, $\text{SD}=2.236$

1. $E(X)=\sum xp(x)=13$. $E(X^2)=\sum x^2p(x)=174$.
2. $\text{Var}(X)=174-(13)^2=5$; $\text{SD}=2.236$.

8. X has support 11, 13, 15, 17, 19 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3. Using $q_p=\min\{x \text{ in support: } F(x)\geq p\}$, find $q_{0.7}$.

Reference: Choose the first support point whose cumulative probability reaches or exceeds p .

Answer: 17

1. The cumulative probabilities are 0.1, 0.25, 0.45, 0.7, 1.
2. The first cumulative probability at least 0.7 occurs at $x=17$.