

Discrete CDFs, Transformations, and Moments

Use the convention $F(x)=P(X\leq x)$. For quantiles, choose the smallest support value whose CDF reaches the target.

1. X has support -3, -1, 1, 3 with probabilities 0.2, 0.3, 0.1, 0.4. For $Y=X^2$, give the PMF and merge repeated transformed values.

Reference: When several X values map to one Y value, add their probabilities.

2. X has mean 10 and variance 7. Let $Y=-3X+3$. Find $E(Y)$, $Var(Y)$, and $SD(Y)$.

Reference: $E(aX+b)=aE(X)+b$; $Var(aX+b)=a^2 Var(X)$; SD scales by $|a|$.

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3. X is equally likely to be 6 or 10; Y is equally likely to be 4 or 12. Compare their means and standard deviations.

Reference: Equal means do not imply equal spread. Compute probability-weighted variance around the common mean.

4. For support 7, 9, 11, 13 with probabilities 0.25, 0.25, 0.25, 0.25, evaluate $F(6)$, $F(10)$, and $F(14)$.

Reference: A discrete CDF is right-continuous and constant between support points.

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5. X has support 8, 10, 12, 14, 16 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3 in matching order. Make a PMF table and a step-CDF table.

Reference: $F(x)=P(X\leq x)$. The CDF is 0 below the support and 1 at or above its largest value.

6. Independent X and Y have $E(X)=11.6$, $Var(X)=2.44$, $E(Y)=2$, and $Var(Y)=2$. Find the mean, variance, and SD of $X+Y$.

Reference: Means always add. Variances add here because independence makes covariance zero.

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7. X has support 10, 12, 14, 16 with probabilities 0.25, 0.25, 0.25, 0.25. Find $E(X)$, $E(X^2)$, $Var(X)$, and $SD(X)$.

Reference: Use probability-weighted sums; $Var(X)=E(X^2)-[E(X)]^2$ and SD is the positive square root.

8. X has support 11, 13, 15, 17, 19 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3. Using $q_p=\min\{x \text{ in support: } F(x)\geq p\}$, find $q_{0.7}$.

Reference: Choose the first support point whose cumulative probability reaches or exceeds p.