Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

1. **X has support -3, -1, 1, 3 with probabilities 0.2, 0.3, 0.1, 0.4. For Y=X^2, give the PMF and merge repeated transformed values.**

Reference: When several X values map to one Y value, add their probabilities.

2. **X has mean 10 and variance 7. Let Y=-3X+3. Find E(Y), Var(Y), and SD(Y).**

Reference: E(aX+b)=aE(X)+b; Var(aX+b)=a^2 Var(X); SD scales by |a|.

State the model or rule, show exact setup, then round only the final decimal.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

3. **X is equally likely to be 6 or 10; Y is equally likely to be 4 or 12. Compare their means and standard deviations.**

Reference: Equal means do not imply equal spread. Compute probability-weighted variance around the common mean.

4. **For support 7, 9, 11, 13 with probabilities 0.25, 0.25, 0.25, 0.25, evaluate F(6), F(10), and F(14).**

Reference: A discrete CDF is right-continuous and constant between support points.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

5. **X has support 8, 10, 12, 14, 16 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3 in matching order. Make a PMF table and a step-CDF table.**

Reference: F(x)=P(X<=x). The CDF is 0 below the support and 1 at or above its largest value.

6. **Independent X and Y have E(X)=11.6, Var(X)=2.44, E(Y)=2, and Var(Y)=2. Find the mean, variance, and SD of X+Y.**

Reference: Means always add. Variances add here because independence makes covariance zero.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

7. **X has support 10, 12, 14, 16 with probabilities 0.25, 0.25, 0.25, 0.25. Find E(X), E(X^2), Var(X), and SD(X).**

Reference: Use probability-weighted sums; Var(X)=E(X^2)-[E(X)]^2 and SD is the positive square root.

8. **X has support 11, 13, 15, 17, 19 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3. Using q_p=min{x in support: F(x)>=p}, find q_0.7.**

Reference: Choose the first support point whose cumulative probability reaches or exceeds p.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

1. **X has support -3, -1, 1, 3 with probabilities 0.2, 0.3, 0.1, 0.4. For Y=X^2, give the PMF and merge repeated transformed values.**

Reference: When several X values map to one Y value, add their probabilities.

Answer: support 1, 9; probabilities 0.4, 0.6

1. Both -1 and 1 map to 1, so P(Y=1)=0.3+0.1=0.4.
2. The outer pair maps to 9, so its probability is 0.2+0.4=0.6.

2. **X has mean 10 and variance 7. Let Y=-3X+3. Find E(Y), Var(Y), and SD(Y).**

Reference: E(aX+b)=aE(X)+b; Var(aX+b)=a^2 Var(X); SD scales by |a|.

Answer: E=-27, Var=63, SD=7.937

1. E(Y)=-3(10)+3=-27.
2. Var(Y)=(-3)^2(7)=63; SD(Y)=7.937.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

3. **X is equally likely to be 6 or 10; Y is equally likely to be 4 or 12. Compare their means and standard deviations.**

Reference: Equal means do not imply equal spread. Compute probability-weighted variance around the common mean.

Answer: both means 8; SD(X)=2, SD(Y)=4

1. Both symmetric pairs have mean 8.
2. SD(X)=2 and SD(Y)=4, so Y is more spread out.

4. **For support 7, 9, 11, 13 with probabilities 0.25, 0.25, 0.25, 0.25, evaluate F(6), F(10), and F(14).**

Reference: A discrete CDF is right-continuous and constant between support points.

Answer: 0, 0.5, 1

1. F(6)=0 because it lies below the support.
2. F(10)=0.5 because only 7 and 9 are at most 10.
3. F(14)=1.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

5. **X has support 8, 10, 12, 14, 16 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3 in matching order. Make a PMF table and a step-CDF table.**

Reference: F(x)=P(X<=x). The CDF is 0 below the support and 1 at or above its largest value.

Answer: CDF values 0.1, 0.25, 0.45, 0.7, 1

1. Cumulative sums at the support are 0.1, 0.25, 0.45, 0.7, 1.
2. Thus F(x)=0 below 8, takes those cumulative values across successive steps, and equals 1 from 16 onward.

6. **Independent X and Y have E(X)=11.6, Var(X)=2.44, E(Y)=2, and Var(Y)=2. Find the mean, variance, and SD of X+Y.**

Reference: Means always add. Variances add here because independence makes covariance zero.

Answer: mean 13.6, variance 4.44, SD 2.107

1. E(X+Y)=11.6+2=13.6.
2. Var(X+Y)=2.44+2=4.44; SD=2.107.

Name _____ Score _____ Teacher _____ Date _____

Discrete CDFs, Transformations, and Moments

Worked Answer Key

Use the convention F(x)=P(X<=x). For quantiles, choose the smallest support value whose CDF reaches the target.

7. **X has support 10, 12, 14, 16 with probabilities 0.25, 0.25, 0.25, 0.25. Find E(X), E(X^2), Var(X), and SD(X).**

Reference: Use probability-weighted sums; Var(X)=E(X^2)-[E(X)]^2 and SD is the positive square root.

Answer: E=13, E(X^2)=174, Var=5, SD=2.236

1. E(X)=sum xp(x)=13. E(X^2)=sum x^2p(x)=174.
2. Var(X)=174-(13)^2=5; SD=2.236.

8. **X has support 11, 13, 15, 17, 19 with probabilities 0.1, 0.15, 0.2, 0.25, 0.3. Using q_p=min{x in support: F(x)>=p}, find q_0.7.**

Reference: Choose the first support point whose cumulative probability reaches or exceeds p.

Answer: 17

1. The cumulative probabilities are 0.1, 0.25, 0.45, 0.7, 1.
2. The first cumulative probability at least 0.7 occurs at x=17.