

Math Sheet

Name _____ Score _____ Teacher _____ Date _____

Density Histograms and the LLN

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

- 1. Repeated iid samples from a population with mean 18.9 produced sample means (add 8.9 to each listed value): $n=10$: 8.7,11.2,9.4,10.8; $n=100$: 9.8,10.2,10.1,9.9; $n=1000$: 10.02,9.98,10.01,9.99. Interpret using the LLN, and distinguish one original observation X from the average $\bar{X}$.**

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

- 2. Bins $[0,1)$, $[1,3)$, $[3,6)$: widths 1,2,3; counts 10,16,15; total 41. Which bin has greatest density?**

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

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Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

3. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,12,18; total 39. Find each relative-frequency density.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

4. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,10,19; total 38. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

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5. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,11,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

6. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 12,13,14; total 39. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

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Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

7. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 8,12,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

8. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,14,14; total 37. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Density Histograms and the LLN

Worked Answer Key

Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

1. Repeated iid samples from a population with mean 18.9 produced sample means (add 8.9 to each listed value): $n=10$: 8.7,11.2,9.4,10.8; $n=100$: 9.8,10.2,10.1,9.9; $n=1000$: 10.02,9.98,10.01,9.99. Interpret using the LLN, and distinguish one original observation X from the average $\bar{X}$.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: Larger- n sample means concentrate near 18.9; one X remains one population draw

1. One original observation X is a single population draw; increasing n does not make that draw itself an average.
2. $\bar{X}$ averages n iid observations, has variance σ^2/n , and the supplied means show decreasing spread around the population mean.
3. This illustrates LLN concentration, not a claim that the sampling shape is normal.

2. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 10,16,15; total 41. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: bin 1, density 0.2439

1. Compare $\text{count}/(\text{total} \times \text{width})$, not raw counts.
2. bin 1, density 0.2439

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Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

3. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,12,18; total 39. Find each relative-frequency density.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: 0.2308, 0.1538, 0.1538

1. $\text{density} = \text{count} / (\text{total} \times \text{width})$.
2. 0.2308, 0.1538, 0.1538

4. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,10,19; total 38. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: 0.2632

1. $\text{area} = \text{count} / \text{total} = 10 / 38 = 0.2632$ (equivalently width times the unrounded density).

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Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

5. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,11,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: density x width = relative frequency

1. Density height is $\text{count}/(\text{total} \times \text{width})$; every bar area is $\text{count}/\text{total}$ and total area is 1.

6. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 12,13,14; total 39. Which bin has greatest density?

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: bin 1, density 0.3077

1. Compare $\text{count}/(\text{total} \times \text{width})$, not raw counts.
2. bin 1, density 0.3077

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Convert counts to relative-frequency density using bin width, read probability from area, and interpret supplied repeated-sampling summaries as evidence of LLN concentration.

7. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 8,12,13; total 33. Explain how count and density heights differ while encoding the same frequencies.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: density x width = relative frequency

1. Density height is $\text{count}/(\text{total} \times \text{width})$; every bar area is $\text{count}/\text{total}$ and total area is 1.

8. Bins [0,1), [1,3), [3,6): widths 1,2,3; counts 9,14,14; total 37. Find the relative frequency in [1,3) from bar area.

Assumptions: Bins are left-closed and right-open; each observation belongs to one bin.

Density equals relative frequency divided by bin width; probability is bar area. LLN summaries use iid samples.

Answer: 0.3784

1. $\text{area} = \text{count}/\text{total} = 14/37 = 0.3784$ (equivalently width times the unrounded density).